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Novice Elementary Teachers' Probabilistic and Statistical Reasoning: Addressing Misconceptions

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Abstract

This study evaluates the probabilistic and statistical reasoning of novice teachers about some basic concepts in statistics. For this purpose, a mixed methodology was used through a quantitative, qualitative approach, in which a non-probabilistic sample of 248 novice teachers was considered. The number consists of 108 men and 140 women from three Colleges of Education. Quantitatively, the study compared the mean reasoning scores of male and female teachers in statistical and probabilistic knowledge using a t-test for independent samples, while qualitatively the comments of the novice teachers were analyzed through content analysis. Among the most relevant findings, there were statistically significant differences between male and female teachers in general statistical and probabilistic reasoning. Likewise, the data provided evidence that novice teachers have equiprobability and representativeness biases. In short, both male and female novice teachers were not able to demonstrate sound reasoning in statistical and probabilistic situations. Professional development programs are needed to support teachers, especially novice ones, in the acquisition of probabilistic and statistical reasoning skills

Keywords: Gender, independent sample t-test, novice teachers, statistics education research, statistics and probability, statistical reasoning

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INTRODUCTION

Statistics education is important for every citizen to face challenges in life (Ubilla et al., 2021). The overall goal of statistics education is to help students develop statistical and probabilistic reasoning to be responsible citizens. In statistics education; posing questions, data collection, data analysis, data representation, probability language, interpretation and inference, and decision-making are important goals of teaching at primary and middle schools. The achievement of these goals depends upon effective teachers who are motivated to develop sound statistical and probabilistic reasoning among students (Estrada & Batanero, 2020). Probabilistic and statistical ideas are everywhere. Learners come to the classroom with knowledge of probabilistic concepts which may be positive or negative. In whichever ways, the knowledge of teachers is essential in strengthening their positive ideas while addressing the negative ideas or misconceptions. Primary and middle school teachers are expected to identify learners' misconceptions and address same (Sharma, 2016). For teachers who will be the first to teach probability and statistics ideas to students formally, their probabilistic and statistical reasoning is important in creating appropriate contexts and raising critical questions to support students' statistical literacy skills.

While several studies have examined pre-service and in-service teachers' probabilistic misconceptions across the globe, few delve into novice teachers' readiness to teach probabilistic and statistical concepts. These few studies were conducted in developed countries. The quality of teachers' mastery of statistical and probabilistic reasoning is considered crucial to addressing misconceptions. This will in turn prepare students to access source bias and the quality of information consumed in public life. For instance, understanding health information and critically assessing newspapers which are sometimes not properly regulated. Damişman and Tanişh (2017) found that the pedagogical content knowledge of high school mathematics teachers on probability was inadequate. This, according to them, would affect the quality of their teaching and how they (teachers) develop sound probabilistic and statistical reasoning in learners.

Teachers' knowledge is key to the development of statistical and probabilistic reasoning. According to Groth (2017), teachers' statistical subject matter is essential together with statistical knowledge for teaching. "The knowledge of the teacher should be studied according to the discipline that (s)he teaches since each knowledge area has its particularities" (Henriques & Oliveira, 2013, p. 2). Shulman (1986) identified three kinds of knowledge (content knowledge, pedagogical content knowledge, and pedagogical knowledge) necessary for developing conceptual understanding in learners. Ozmen and Baki (2021), argued that to raise statistically literate citizens, an investigation into teaching statistical and probabilistic reasoning is essential in schools and colleges or universities at all levels. This argument is sustainable and we contend that such investigation is necessary for novice teachers at primary and middle schools.

How concepts are introduced to a child is key to the child's intellectual growth in that area. Teachers can't produce effective and efficient citizens who can make informed choices if their statistical and probabilistic reasoning is inadequate. Shulman (1986) argued that the content knowledge of teachers is important in the implementation of the curriculum. Content knowledge includes concept knowledge, ideas, knowledge of proofs, and assessment of evidence. All these elements are crucial in teaching and learning statistical and probabilistic concepts, particularly in addressing misconceptions and promoting statistical reasoning. For teachers to alleviate the misconceptions of students they must design instructional strategies to allow students to confront their biases or misconceptions (Sharma, 2016; Hokor, 2023). Evidence-based teaching and proofs are key for promoting statistical and probabilistic reasoning.

Hokor (2020) proposed several activities for addressing probabilistic misconceptions, particularly in the area of equiprobability bias, representativeness bias, and positive and negative recency effects. The equiprobability bias is the thinking that every random event has an equal chance of occurrence. For example, people with equiprobability bias think that when we have 5 white balls and 7 yellow balls in a container, the possibility of picking a white ball without looking is the same as picking the yellow ball. However, the yellow ball has a greater chance of being picked as compared to the white ball since we have more yellow balls than white. Representativeness misconception is "the tendency of students to erroneously think that samples which resemble the population distribution are more probable than samples which do not" (Anway & Bennet, 2004, p. 1) without consideration of the sample size. People with representativeness bias hold the view that in the toss of a coin four times, the sequence THTH is more likely than HTHH because it resembles the population (Hokor et al., 2022). Nonetheless, they are all equally likely. Another example is the roll of a die six times and seeing the sequence 241653 as more likely than 466361. On the contrary, people with outcome approach bias would also answer equally likely (Konold et al., 1993). The implication for justification offered by some participants on these types of sequences, "one cannot predict with assurance which sequence will occur" (Konold et al., 1993, p. 400) is consistent with the outcome approach. This is where the chance of the occurrence or nonoccurrence of an event is treated as an affirmation of certainty instead of a measure of possibility (Belova & Zowada, 2020).

Positive and negative recency effects are other common misconceptions. When one tosses a coin and obtained an outcome and thinks that outcome has a direct influence on the next outcome. For example, when you roll a die three times and obtained the sequence 444, the thinking that in your next toss, you are more likely to get a 4 is termed as positive recency effect (Hokor, 2020). However, when one thinks that the other numbers on the die are more likely to occur than a 4, it is considered a negative recency effect (Hokor et al.). Each roll of a die is independent as such the numbers have the same chance of occurrence. Konold et al. (1993) investigated students' reasoning using four possible sequences of the “most likely” outcome from tossing a coin number of times. Seventy-two percent of the participants correctly answered that all the sequences are equally likely. Surprisingly, only half of the participants who answered correctly on the “most likely” answered correctly again on the “least likely”.

While much is known about students acquiring probability misconceptions in informal settings and bringing the same to classrooms, and activities to address misconceptions proposed, little is known about novice teachers’ probabilistic and statistical reasoning and how this reasoning could help address misconceptions. The novice teachers’ initial statistical and probabilistic reasoning knowledge identification can inform training or in-service programs to adequately implement the teaching of statistics and probability concepts. Novice teachers are teachers who have taken all the courses prescribed in their programs of study and are about to start practicing. Aseeri (2015) recommended that professional development programs should pay more attention to teachers' needs. In light of this, Slama et al. (2021) state three goals researchers should focus on for better professional development programs:

1. understand the state of mathematics teaching and learning,
2. define the problem and opportunities for supporting mathematics teacher learning,
3. and develop concept-specific tasks to enhance mathematics teacher knowledge and skills.

We argue that identifying novice primary and middle school teachers’ statistical and probabilistic reasoning abilities will play an important role in in-service professional development programs in terms of task design and support. This study sought to assess novice teachers’ probabilistic and statistical reasoning and how they can address probability and statistics misconceptions In line with this purpose, the following research questions were considered.

1. Do male and female novice teachers’ statistical and probabilistic reasoning scores differ, and what patterns of reasoning exist in their responses?
2. What patterns of reasoning emerged from novice teachers’ responses to equiprobability bias tasks after taking all the required statistics courses in their training?
3. What patterns of reasoning emerged from novice teachers’ responses to representativeness tasks after taking all the required statistics courses in their training?

This study gives us a better view to anticipate probability and statistics instructions of novice teachers. Additionally, it provides statistics educators the feedback or assessment of their teaching strategies on their pre-service teachers. This can lead to instructional reforms in statistics education. For example, in mixed (both male and female) classrooms the study may inform teachers in the selection of types of activities in addressing probabilistic and statistics misconceptions. Also, the nature of in-service program activities about probabilistic and statistical reasoning development of teachers will significantly be impacted.

LITERATURE REVIEW

Probabilistic and Statistical Reasoning

Statistics, as authors such as Groth (2007) have mentioned, has its object of study that distinguishes it from mathematics. In this context, we can talk about statistical reasoning, which is a particular type of reasoning associated with statistical concepts. Statistical reasoning is associated with the ability to connect different ideas and cardinal concepts of statistics and or probability, to understand and explain these processes (Garfield & Ben-Zvi, 2008) and make explicit the obtaining of a result. It also constitutes the selection of a particular model and the justification of why it reasonably adjusts to the situation studied (delMas, 2004). Probabilistic and statistical reasoning refers to how people reason with statistical and probabilistic ideas and make sense of probabilistic and statistical information (Garfield & Gal, 1999). People with probabilistic and statistical reasoning skills understand, interpret, and analyze statistical information and uncertain situations for the best possible way of acting or making decisions in the interest of the public and oneself. However, Borovcnik and Kapadia (2018) noted that no model allows the evaluation of statistical reasoning in teachers. Some studies examined the statistical reasoning of students.

Liu and Garfield (2002) when examined gender differences in the statistical reasoning of 245 college students from the United States and Taiwan on both correct reasoning scores and misconception scores. They found statistically significant mean differences between males and females statistical reasoning in favor of male students. However, they found no mean differences in statistical reasoning scores between males and females for the two countries. Also, Ortiz and Alsina (2019) conducted an exploratory study on pre-kindergarten children aged 4 to 6 who had not formally studied probability ideas about chance and probability. They found that children use basic probabilistic language and had informal knowledge about sample space, the possibility of occurrence, and the comparison of probability. While recommending visual support or concrete materials in the teaching context for the introduction of probabilistic notions to first-time learners, they also called for continual exploration of what contributes to Early Childhood Education teachers' acquisition of probabilistic literacy and practices.

Rodriguez-Alveal et al. (2022) used real-world problems of uncertainty to investigate pre-service mathematics teachers' and in-service mathematics teachers' literacy and probabilistic thinking skills. Their study of 55 trainees and in-service teachers found that the participants have not developed probabilistic thinking skills to deal with problems based on classical and frequency perspectives and the classical meaning of probability. This suggests that the participants will not be able to develop sound probabilistic reasoning and literacy among students through instruction. This is because a teacher's content knowledge plays a significant role in his/her instruction.

The investigations by Martin et al. (2017) on the effects of gender on statistical reasoning while controlling for experience and individual differences saw the performance of both male and female participants improved but males demonstrated better reasoning than females. Also, Henriques and Oliveira (2013) examined in-service teachers' knowledge of statistical investigations and the ability to develop statistical thinking in students. Their findings portrayed that some components of statistics need more attention. For example, in-service teachers resort to making general statements about their anticipated actions instead of defendable arguments. The gender differences in statistical achievement among 18 undergraduate students of postsecondary-level psychology, education, and business courses were examined (Schram, 1996). The result was mixed. While univariate tests showed that males performed better than females on the outcome of a series of exams, females significantly outscored males in the total course performance. Previous research has examined male and female students' statistical and probabilistic reasoning, but none seem to have considered male and female novice teachers.

Equiprobability and Representativeness Misconceptions

Huerta (2020) mentions equiprobability and representativeness biases as present in the problems associated with gambling, taking into account the epistemological roots of the concept of probability. It should be noted that there are different conceptions of the concept of randomness, one of which is randomness as equiprobability, which is common in textbooks (Rodríguez-Alveal et al., 2022).

Investigations such as Gürbüz et al. (2012) on 5^{th} - 8^{th}-grade pupils in comparing and evaluating their probability misconceptions showed that as the level of education increased, the percentage of the correct answers together with the misconceptions increased. Language, beliefs, and experience influence students' informal knowledge of probability (Amir & Williams, 1999). Amir and Williams view language, belief, and experience as an element of culture. The participants in their study were consistent both in the interview and the questionnaire on outcome approach, representativeness bias, availability, and equiprobability bias. Similarly, Hirsch and O'Donnell (2001) found that students with formal statistics and probability education continue to demonstrate misconceptions. These findings have implications for novice teachers since they are the ones who are going to take charge of the responsibility of correcting these misconceptions in their learners.

Paul and Hlanganipai (2014) investigated high school students' gender differences in probability misconceptions across three grades (10, 11, and 12) and noted differences in favor of male students. This means that male students reported fewer misconceptions compared to their female counterparts. While there are mixed reports on the sex differences in mathematical achievement in many areas of mathematics (trigonometry, geometry, algebra), the evidence appears to be inadequate in the area of probabilistic and statistics reasoning of novice teachers. This current study examined the mean differences between male and female novice teachers' probabilistic and statistical reasoning mean scores.

Statistics Education

In developed countries, statistics is taught as a subject right from elementary school to tertiary (Ozmen & Baki, 2021). For example, statistics has its syllabus in Chile and Spain both in primary education, which is compulsory education and in teacher training institutions (Ubilla et al., 2021). However, the story is not the same in developing countries. In some African countries, for instance, statistics is only considered a subject of study from high school through tertiary. In Ghana, statistics is considered as a subject of study only at the tertiary level. Nevertheless, statistics and probability is included in the mathematics curriculum for primary and middle schools in Ghana.

Primary and middle school teachers are expected to expose learners to probabilistic languages (not possible, possible, less likely, equally likely, highly likely, certain) (Ministry of Education, 2019a) and also list possible outcomes of coin tossing, rolling a die, spinners, and finding the probability of simple events through game activities (Ministry of Education, 2019b). Additionally, they are required to teach primary school pupils how to collect data, represent data graphically, and interpret data meaningfully (Ministry of Education, 2019b). Ubilla et al. (2021) argue that several teachers did not receive adequate training to teach statistics and probability concepts and this would affect the quality of instruction.

Statistics and Probability Curriculum in Colleges of Education (CoEs) in Ghana

The Mathematics Curriculum for CoE, implemented by college tutors in training pre-service teachers to teach at primary and middle schools, was designed considering what they are going to teach. Considering this, pre-service teachers at the Colleges of Education who offered Bachelor of Education in Primary Education took Statistics and Probability I, while those who offered B. Ed in Junior High School Mathematics and ICT Education took Statistics and Probability II. Statistics and Probability II covered additional areas such as normal and t-distributions, inference for means, permutation, and combination than Statistics and Probability I. Both groups took Educational Statistics course in addition to the above courses.

These statistics courses are offered in the second semester of the third year. The curriculum recommends empirical and practical work in probabilistic and statistical education around the globe to provide the appropriate context for pre-service teachers' statistical and probabilistic reasoning development (University of Cape Coast, 2021). These statistics curriculums are, therefore, intended to equip pre-service teachers with the knowledge, skills, and values needed to teach statistics and probability to Primary and middle school pupils in everyday life context. The areas that the novice teachers were exposed to in the statistics and probability courses are; collection, organization, representation, analysis, and interpretation of data, measures of central tendency, dispersion, experiments and probability of simple events, and conditional probability. Additionally, the Educational Statistics course focused more on inferential statistics. Some of the areas are hypothesis testing, measures of relationships, and simple regression. These courses in the CoE curriculum further address the professional development of pre-service teachers of statistics and highlights differences between statistics and mathematics that have important implications for teaching and learning (University of Cape Coast, (2021). These courses also provide real-life experiences or context for using statistical and probabilistic ideas and the requisite resource material for preparing pre-service teachers to teach statistical and probabilistic concepts sufficiently and effectively in primary and middle schools.

The teaching philosophy that was prescribed in this curriculum is the socio-constructive approach where tutors are required to consider the various background of their learners and design appropriate tasks that stimulate learning. The teaching approaches aligned with the social constructivist theory of Vygotsky (1978) that learning should be a social activity. This form of learning empowers learners to reconstruct concept or reorganize new ideas with existing ones and apply it to solve problems in new situations or real life (Hokor, 2022). Learners in constructivist classrooms are responsible for their strategies under the supervision of a teacher or tutor.

METHOD

Design

To better understand novice teachers' reasoning both in terms of computations and conceptual understanding of the concepts under study, the researchers employed the mixed methods approach as a means of inquiry. That is through quantitative and qualitative data, this study examined the probabilistic and statistical reasoning of novice teachers and their ability to address misconceptions. According to Creswell (2012), one conducts mixed methods research when one design, qualitative or quantitative research, is not enough to find answers to all the research questions in a study or find an alternative viewpoint to study. With this in mind, the researchers particularly employed the embedded mixed methods design, where quantitative data and qualitative data were both collected at the same time with one serving as a supplement (Creswell, 2014). The qualitative data was embedded in the quantitative data to provide a better understanding of novice teachers' statistical and probabilistic reasoning and the extent to which they can address misconceptions. In line with this, the Probabilistic and Statistical Reasoning Assessment (PSRA) test, used in this study, has two parts: multiple-choice and justification. The multiple-choice part was scored and classified as quantitative data while the justification parts were treated as qualitative data.

Participants and Selection

The participants were selected from three CoEs (denoted by A, B, and C) in two regions in the Republic of Ghana. These three colleges are affiliated with the University of Cape Coast as such implement the same curriculum. The number of respondents in this study was 248 first batch of novice Bachelor of Education (B. Ed) teachers for primary and middle schools. These participants were involved in the study at the end of their four-year bachelor of education studies. The respondents were selected by non-probability sampling (McMillan & Schumacher, 2011) based on their availability and willingness to participate in the study, while the three Colleges of Education were selected for their proximity to the researchers. Tables 1a and 1b provide college-based and general demographic information respectively of respondents.

Table 1a

College-based Demographic Information of Respondents

	Category	Number per College			Total
		A	**B**	**C**	
Age	18 – 23	21 (20.6%)	7 (6.9)%	74 (72.5%)	102
	24 – 29	42 (38.9%)	10 (9.3%)	56 (51.8%)	108
	30 – 35	6 (30.0%) n=69	4 (20.0%) n=21	10 (50.0%) n=140	20 n=230
Sex	Male	0	9 (8.3%)	99 (91.7%)	108
	Female	70 (50.0%) n=70	15 (1,.7%) n=24	55 (39.3%) n=154	140 n=248
Program of Study	Primary Education	70 (31.3%)	154 (68.7%)	0	224
	JHS Education	0 n=70	0 n=154	24 (100.0%) n=24	24 248

Table 1b

Demographic Information of Respondents

		Male (n=108)	Female (n=140)	p-value
	Age (years)	24.3±2.9	24.1±2.9	0.56
Programme	Primary Education	93 (86.1%)	131 (93.6%)	0.04
	JHS Education	15 (13.9%)	9 (6.4%)	

The respondents consist of 108 males representing 43.5% and 140 females representing 56.5% of the sample (see Table 1b). Out of this number, 70 (0 males and 70 females) were selected from College A, 24 (9 males and 15 females) were selected from College B while the majority, 154

(99 males and 55 females were selected from College C (see Table 1a). Regarding age, both male and female groups have a very similar average age (M=24.3 years), with no statistically significant mean differences. Also, the majority of the participants fall within the age range of 24-29 years, representing 46.9%.

Test instrument

The Probabilistic and Statistical Reasoning Assessment (PSRA) is made up of both items on probability and statistics. These items were designed and validated in the previous research to measure the PSRA of pre-service teachers (Hokor et al., 2022; Hokor & Sedofia, 2021; Garfield, 2003). Their study reported a KR20 reliability coefficient of 0.824. However, the current study found reliability of .703 There are 13 items on the questionnaire and each correct response was scored 1 point else 0, with a total score ranging from 0 to 13 across all items. The items were both closed-ended and open-ended. The probabilistic items were adopted and modified from the items used in the studies (Hokor & Sedofia, 2021; Hokor et al., 2022).

The modification to the items was the demand for reasons or justifications for the choice of answers as the study sought to ascertain the ability of novice teachers to develop probabilistic and statistical reasoning in their prospective learners. Also, to provide further information on novice teachers’ probabilistic and statistical reasoning. This means the novice teachers’ evidence of statistical and probabilistic reasoning is a crucial indicator of their ability to develop the same in their learners.

The statistical reasoning items (2, 4, and 6) were adopted from (Garfield, 2003). There was a follow-up on item 6 which seeks justification for the chosen answer in 6. This was the only item modified from the work of (Garfield, 2003) because the items request the respondents to choose among interpretations that best explain the statistical statements. The items in the PSRA relate to the concepts of sample space, probability languages, probability of an event, probability reasoning, and interpretation of statistical and probabilistic information. These are the basic concepts or ideas in the primary school curriculum (Ministry of Education [MoE], 2019a; 2019b). Gómez-Torres et al. (2016) argued that the assessment of novice teachers' probability and statistics knowledge for teaching should take into consideration what they are going to teach at that level. Addressing probability misconceptions in students is an important goal of elementary statistics education. Table 2 considers some probability reasoning bias and statistical interpretations with its associated items.

Table 2

Components/bias and Corresponding Items

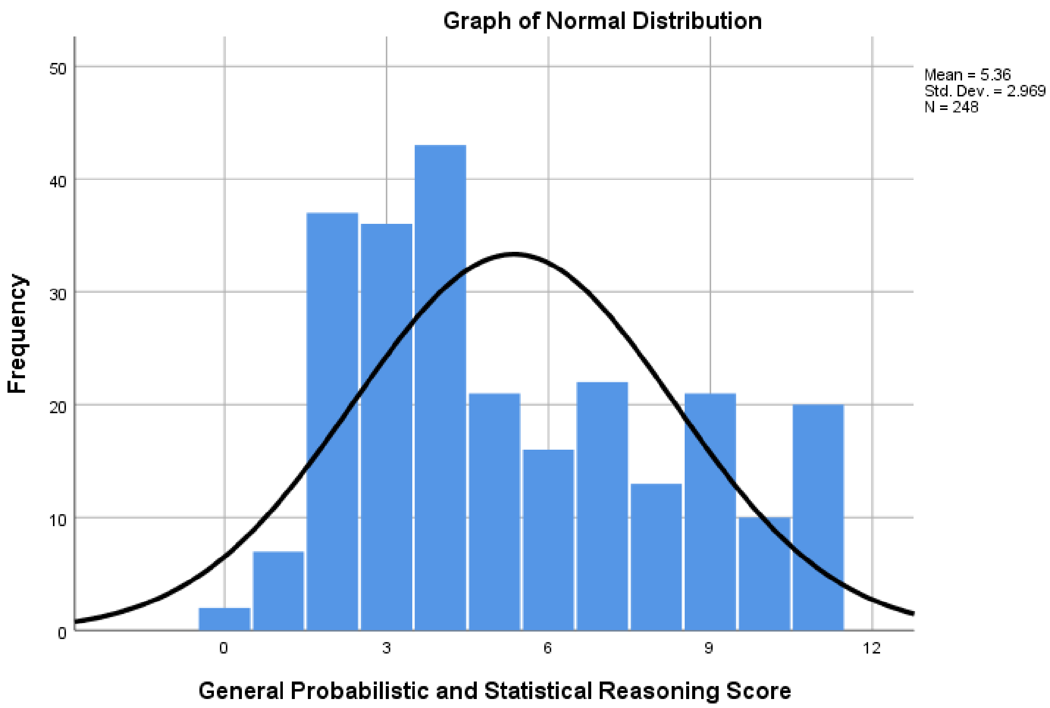
Component/Bias	Item
Equiprobability Bias	3, 9, 11, 12
Representativeness Bias	1, 7, 8, 13
Statistical and Probabilistic Reasoning Competence	2, 4, 5, 6, 10

Data Analysis

All the participants were made to take a test at the end of their four years of studying at college to ascertain their preparedness to teach probability and statistics units. The data, obtained from the test, were analyzed using independent samples t-test to determine significant mean differences between men and women in probabilistic and statistical reasoning since the data collected is on a continuous scale (Creswell, 2014). The scores were from two different groups (male and female). Before running the independent samples t-test, we checked the assumption of normality. The scores appear to be normally distributed from the graph next.

Figure 1

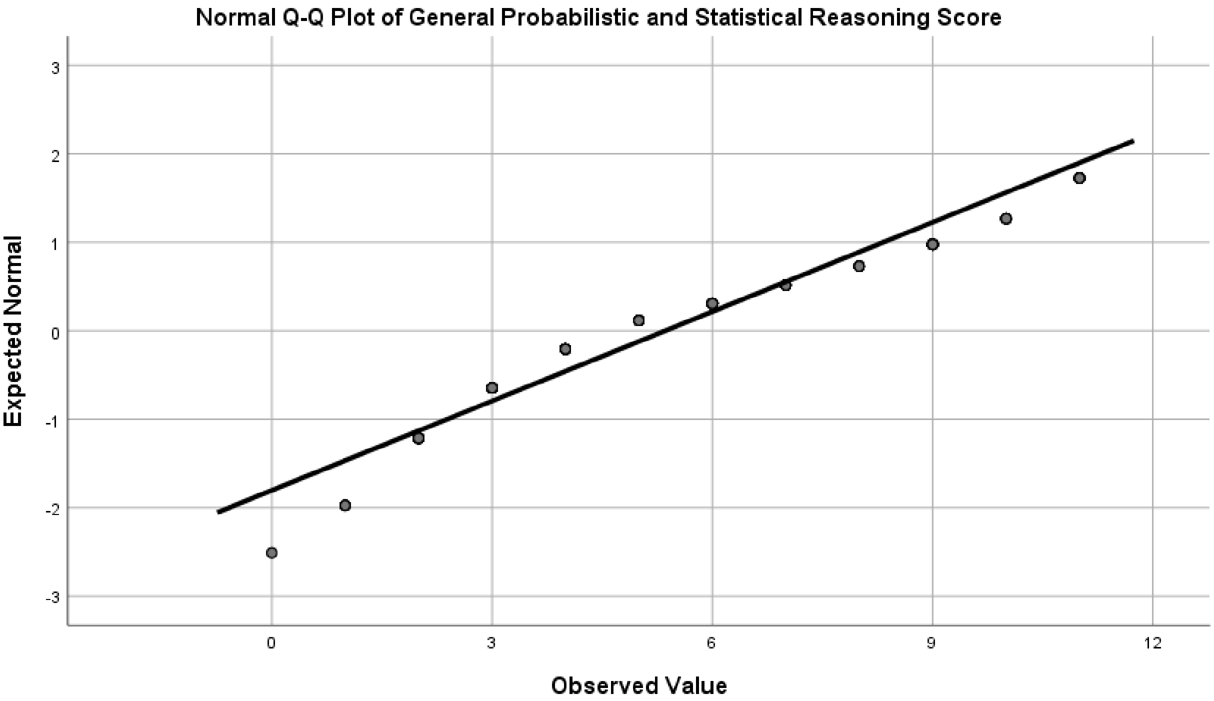
Graph Showing The Normality of Scores



The Q-Q plot of Figure 2 below revealed that the novice teachers' statistical and probabilistic reasoning scores (dependent variable) point to the normality of the independent variables as many of the values were close to the straight line.

Figure 2

Normal Q-Q Plot Showing Probabilistic and Statistical Reasoning Scores



Levene's test was also conducted to check the equality of variances. Table 3 presents the results of the test.

Table 3

Results of the Levene Test

Levene Statistic	Df1	Df2	Sig
.174	1	246	.677

The variances between male and female novice teachers are not significantly different since the p-value = 0.677 is greater than 0.05. This provides legitimacy for conducting independent sample test.

Additionally, the qualitative data collected was analyzed through verbal quoting of some participants to provide more insights into new teachers' probabilistic and statistical reasoning acquisitions. Content analysis was done on the qualitative data (Krippendorf, 1997). The clarity of justifications was considered on which one to include as qualitative data. Both correct and wrong reasons given by the respondents were shared in this regard for the purpose of education. The patterns of reasoning in the responses of the participants emerged from the analysis which provide more insight into their thinking. The content analysis helped to describe treads/patterns in novice teachers' reasoning.

To protect the identity of the participants, codes such as PR4922F, PR5044F, JH335F, JH9933M, and JH14233M were used to represent each respondent. The PR begins with the participants who offered B. Ed in Primary Education while JH begins with those who offered B. Ed in JHS Mathematics and ICT Education. If the respondent's number ends with M, then the person is a male. Similarly, when it ends with F, then the person is a female. Also, the last two digits, (22, 33, or 44), before the gender of a participant represents whether the respondent is from college A, B, or C. For example, the code PR2322M would mean a male respondent from college A who offered B. Ed in Primary Education. The data obtained from the PRSA was analyzed based on the research questions that guided the study. Furthermore, the qualitative data were classified under correct reasoning and indicators of misconceptions.

RESULTS

Research Question One

This research question investigated whether or not, male and female novice teachers' probabilistic and statistical reasoning differ in the areas assessed. To achieve this, an independent samples t-test was conducted at a significance level of 5% to test the mean scores of male and female respondents in all areas measuring their probabilistic and statistical reasoning knowledge. The result is illustrated in Table 4.

Table 4

Gender Differences Between the Mean scores of the Probabilistic and Statistical Reasoning of the Participants

	Female (n=140)	Male (n=108)	p	Effect Size
Probabilistic and Statistical Reasoning	5.01±2.94	5.81± 2.96	0.03	0.02

The results revealed that there was statistically significant mean differences between the mean scores of the female respondents ($M = 5.01$, $SD = 2.937$) and the male respondents $M = 5.81$, $SD = 2.961$; $t\,(246) = -2.140$, $p < 0.05$ (see Table 4). This result shows that male novice teachers have more knowledge of probabilistic and statistical concepts than females. However, the effect size calculated as a result (d = 0.02) revealed that this mean difference is small.

The result also revealed that both females and males scored below 6.5 marks which is the average mark. This is an indication that the novice teachers' knowledge of probabilistic and statistical concepts is below average. Table 5 shed more insights on the quantitative result with the justifications provided by both males and females on items 6, 5, and 10. These items were considered because of the difficulties they posed to both groups and understanding how participants reason is key to activity designs and instructions.

Table 5

Participants' Probabilistic and statistical reasoning justifications

The justification given by male	The justification given by female
Correct reasoning	
JH4933M: Q5(C): Tossing coin deals with probability. So, when you toss a coin, the probability of getting a head and a tail is the same.	JH533F: Q10(C); The sample space is 2. Calculating the probability of obtaining heads, simply involves dividing the number of heads to be gotten in a single toss that is 1 by the sample space to give $\frac{1}{2}$.
PR1122M: Q10(C): The probability of the coin showing up head is $\frac{1}{2}$ because the previous outcome does not affect the outcome on the next toss.	JH5233F: Q5(C): Getting two heads first does not change the probability of getting a head or a tail on the next toss.
	PR7144F: Q5(C): A coin has only two faces and both faces have equal chances of occurring.
Justifications provided that indicate the presence of misconceptions	
JH9933M: Q6(A); For every newborn girl, Hospital A gets 5 girls and Hospital B gets one girl which means on a particular day, Hospital A is likely to record more female births than Hospital B.	JH233F: Q6(A); Because hospital A's birth rate is higher than that of hospital B.
JH4933M: Q6(C); Hospital A records an average of 50 births a day and half (i.e., 25) are girls. Hospital B records an average of 10 and half (i.e., 5) are girls. Since, 25>5, Hospital A is more likely to record 80% more females.	JH335F: Q6(A): The sample space in hospital A is large, hence the probability of half of all newborns being girls and the other half being boys is realizable unlike in hospital B where the sample space is small, hence less likely to record 50-50 births of boys and girls.
JH15033M: Q10(E): a coin has a head and a tail so getting heads three times in a row is difficult. So, the probability of obtaining a head on the next row is zero.	PR15444F: Q10(A): Head can still appear because the probability of obtaining a head is 1.
PR944M: Q6(A): Both of the hospitals have the maximum chance to record such a percentage but hospital A has a higher chance since its record is higher than hospital B	PR9422F: Q6(A): Considering hospital A, if the total number of birth is 50 on average, and half of 50 girls gives 25. Meaning 80% of girls for hospital A is 20. And in hospital B if the average birth is 10 and half of the 10 is 5 which are girls. 80% of girls in hospital B is 4. Therefore, hospital A records more female births than hospital B.
PR1144M: Q5(C): Because the coin has only two faces, we cannot determine whether the tail or the head will appear.	PR9622F: Q6(C)): The two hospitals are equally likely to record such an event because you can't predict the birth of hospital A or hospital B.
PR1144M: Q10(E): Because there is no guarantee that any of the answers will be obtained hence the probability is zero.	PR6922F: Q6(C); The two hospitals are likely to record the same event because the hospitals are equally likely to record newborns which half are girls and half are boys.
PR11544M: Q5(A): This is because hospital A is said to have more maternity cases than hospital B hence hospital A is more likely to record 80% or more.	PR5722F: Q5(A): This is because both head and tail have equal chances of occurring. Hence, if head has occurred twice, then its chances of occurring again has reduced.
PR14744M: Q6(A): Both hospital A and B record the same sex but hospital A records higher births a day so they are more likely to record more female births than hospital B.	PR1944F: Q6(A): Both hospitals have the maximum chance but hospital A has the highest chance since its number of births per day is higher.

The participants in Table 5 demonstrated different types of reasoning, including correct probabilistic and statistical reasoning, and reasoning that indicated misconceptions or errors in thinking. Table 4 shows that both males and females had difficulty interpreting probability statements. The response to item 6 in particular was surprising as almost all the participants' justifications did not reveal a conceptual understanding of small and large samples. The difficulty encountered may be due to much focus on computational procedures in statistics classrooms instead of real-life or daily use of probability. The interpretations of the responses of the respondents on item 6 do not take into account the small nature of the sample size involved. However, the participants demonstrated some level of knowledge concerning sample size in general but not specific to this context. Correct reasoning was demonstrated through the use of appropriate mathematical concepts and principles to arrive at accurate conclusions. For example, JH4933M and PR1122M in Q10(C) correctly state that the probability of a coin showing up heads is $\frac{1}{2}$, and JH5233F in Q5(C) correctly states that the probability of getting a head or tail on the next toss is not affected by previous outcomes.

On the other hand, the reasoning that indicated misconceptions or errors in thinking was demonstrated through the use of flawed logic, incomplete understanding of concepts, and inaccurate assumptions. These types of reasoning highlight areas where participants may need further education or support to develop a more accurate understanding of probabilistic and statistical concepts. Some responses indicate the presence of misconceptions. For example, JH15033M in Q10(E) incorrectly states that the probability of obtaining a head on the next toss after getting three heads in a row is zero, and PR1144M in Q5(C) incorrectly states that because a coin has only two faces, we cannot determine whether the tail or the head will appear. Similarly, some responses show a lack of understanding or confusion about the concepts being tested. For example, JH9933M in Q6(A) and JH233F in Q6(A) both provides incorrect justifications for why Hospital A is more likely to record more female births than Hospital B, while PR11544M in Q5(A) incorrectly assumes that hospital A is more likely to record 80% or more of all births. Overall, the responses suggest that while some participants have a good grasp of probabilistic and statistical reasoning, others may benefit from further instruction and clarification of these concepts.

Research question two

Research question two investigated the patterns of reasoning of novice teachers on equiprobability bias. Table 6 presents preservice teachers' reasoning on equiprobability bias.

Table 6

Male and female novice teachers' reasoning on equiprobability bias problems

Male's justification	Female's justification
Correct reasoning	
JH4933M: Q11(D); 4 is most likely to occur since it appears twice on the die. Therefore, the probability of 4 occurring is higher than the rest of the numbers.	JH533F: Q11(D); All the other numbers have equal chances of occurring to be $\frac{1}{6}$. But the chance of getting a 4 is $\frac{1}{3}$ which is far greater than that of each of the other numbers.
JH9933M: Q12(B): The chance of obtaining a sum of 4, 7, and 11 are 0.083, 0.167, and 0.056 respectively. Since the sum of obtaining 7 has the greatest chance, it is the most likely outcome.	PR1122F: Q3(B): red because, looking at the number apportioned to both colors, red has more frequency compared to white.
PR11544M: Q3(B): This is because red has the most occurring number on the spinner and hence has more chances of occurring than white.	
Justifications provided that indicate the presence of misconceptions	
PR15044M: Q12(D): All the sums of numbers can be obtained.	JH5233F: Q12(D); Since the dice are fair. It is possible to obtain any of the options.
PR10344M: Q3(C): because the probability that red and	PR9422F: Q3(C): Regardless of the number of colours of a

white can occur is high and both have equal chances of occurring. PR14744M: Q9(C): The probability can be obtained in both sums because any number can be obtained unpredictably	particular part, both white and red are equally likely to occur. PR1522F: Q9(C): Since the two fair dice are to be tossed once. Is likely that a sum of 11 and a sum of 10 are both equally likely. PR9344F: 9((C): because there is a probability of obtaining a 6 on one die and a 4 on the other die. There is also a probability of obtaining 5 on one die and 6 on another die PR6422: Q11(F): They are all equally likely to appear since the die has all the numbers on it. So, when thrown only once, there is a possibility that any of these numbers would occur. PR10044F: Q9(C): All numbers will appear on the die and all sums can be obtained.

From the participants' responses in Table 6, we can identify patterns of correct and incorrect reasoning. Some participants showed correct reasoning by understanding the concept of equiprobability, where all outcomes have an equal chance of occurring. For example, JH4933M and JH533F reasoned correctly by identifying that each number on the die has an equal probability of occurring, while JH9933M and PR1122F reasoned correctly by identifying the most likely outcome based on the probabilities of the possible sums. These participants are more likely to take actions with appropriate tasks to develop sound reasoning in students. However, some participants showed misconceptions in their reasoning in the following areas.

Equiprobability Bias

Some participants assume that all outcomes have equal chances of occurring, which is not always the case. For example, JH533F incorrectly states that the chance of getting a 4 is $\frac{1}{3}$, while the correct probability is $\frac{1}{6}$ since each number has an equal chance of appearing on a fair die.

Misunderstanding of Probability

Some participants seem to misunderstand the concept of probability, as seen in PR10344M and PR9422F's responses. They incorrectly state that both red and white have an equal chance of occurring, when in fact, the probability of each color appearing depends on the number of sections allocated to each color on the spinner.

Generalization

Some participants generalize that all numbers or sums can be obtained with equal probability, which is not always true. For instance, PR15044M's response to Q12(D) assumes that all sums of numbers can be obtained with equal probability, which is not true as some sums have a higher probability of occurring than others.

Research question three

Research question three focused on the patterns of reasoning of novice teachers in representativeness bias tasks. Table 7 presents their justifications for the chosen answer by both males and females to identify patterns of reasoning.

Table 7

Male and female novice teachers' reasoning on representativeness problems

Reasons given by male respondents	Reasons given by female respondents
Correct reasoning	
JH9933M: Q1(G); Because the die has six faces and all the faces have the chance to occur. So, any number can occur.	JH12033F: Q13(D): The outcomes can be THTH, TTTH, or HHHH. So, any of them is likely.
Justifications provided indicate the presence of misconceptions	
JH15033M: Q13(A): The number of times THTH occurs is more than the number of times both TTTH and HHHH occur.	JH334F: Q1(C): Tossing a die is a probability of an event occurring. So, since the die rolled five times brought an outcome of different numbers (51246) there is a possibility that a different number 3 will appear in the next roll.
PR7344M: Q8(C): There is a probability of getting a head in each toss and 1 times 4 is equal to 4.	PR1522F: Q8(C): All head at once is least likely to occur. Because one way or the other a tail might occur.
PR5744M: Q7(B): Any number from 123456 can appear at any time the die is thrown.	PR8622F: Q7(B): 123456 are most likely to occur because each of the numbers has an equal chance of occurring when it is being tossed at any moment or time.
PR11544M: Q7(D): The probability of obtaining the answer in D is higher as compared to the other numbers.	PR1522F: Q7(B): This is because the die is having 1 to 6 if the die is tossed six times, you can have all the numbers.
PR11544M: Q1(C): The probability of obtaining 3 will be greater than the other numbers.	PR2044F: Q1(A): One is directly opposite to 6 so if you roll the die it will turn upside down and the number that will appear is 1.
PR5744M: Q13(A): There is a chance either side will appear.	PR14944F: Q13(B): TTTH is likely to occur due to the four consecutive times the coin is tossed.
PR8244M: Q8(A): The probability is that each side can appear twice.	PR8444F: Q8(C): There is a probability of getting a head in each toss, hence H will be obtained in all four tosses and the result will be HHHH.

Table 7 shows that both male and female teachers correctly identify that, in situations where there are equally likely outcomes, any outcome is equally likely to occur. For example, JH9933M correctly reasons that any number can occur when rolling a die because all faces have the same chance to appear.

However, some male and female teachers demonstrate a misunderstanding of probability by assuming that past outcomes affect future outcomes. For example, JH334F believes that a different number 3 is more likely to appear on the next roll because the die has already rolled five different numbers. Also, the teachers demonstrate a misunderstanding of the representativeness heuristic by assuming that certain outcomes are more likely to occur based on their perceived frequency or similarity to a pattern. For example, JH15033M believes that THTH is more likely to occur than TTTH or HHHH because he thinks it has occurred more frequently in the past. Similarly, PR11544M thinks that the number 3 is more likely to appear than other numbers, while PR5744M believes that any number from 123456 is equally likely to appear at any time. PR14944F also believes that TTTH is more likely to occur than other sequences because it resembles a pattern.

PR5744M and PR8622F think that each of the numbers 123456 has an equal chance of occurring when a die is tossed at any moment or time. While it is true that each number has an equal probability of $\frac{1}{6}$ of appearing on a single toss, the statement suggests that the order in which the numbers appear is not relevant, which is correct. PR11544M believes that the probability of obtaining the answer in option D is higher compared to the other numbers in Q7, without providing any justification for this claim.

DISCUSSIONS

This study examined novice teachers' knowledge of general probabilistic and statistical reasoning for addressing misconceptions at elementary school.

Novice teachers' Statistical and Probabilistic Reasoning Development

The data analysis above showed that there are mean differences in novice teachers' statistical and probabilistic reasoning abilities in terms of gender. Similar results were found by Martin et al. (2017), and Liu and Garfield (2002) in probabilistic and statistical reasoning achievement in gender in favor of males. The only situation where Martin et. al. found no significant mean difference was among women with two statistics courses and men with no statistics course. This suggests that males are ahead of females in statistical reasoning before the introduction to statistical courses.

Nonetheless, both groups (male and female) in this study scored below the average in general statistical and probabilistic reasoning. One reason for this may be due to a lack of attention to the functional demands of statistics in the real world when designing a curriculum for statistical and probabilistic reasoning since some of the items were focused on real-life contexts. It pre-supposes that, most tutors tend to focus more on procedural knowledge rather than real-life applications which has implications for students' acquisition of statistics and probability reasoning skills. Alternatively, the curriculum may adhere to the functionality of probability and statistics in real life but teaching and learning may not meet the functional demands of probabilistic and statistical concepts in the real world. And this would affect the teacher's interpretation of probabilistic and statistical information/statements within context. Similarly, the findings suggest that teachers' knowledge of basic statistical and probabilistic concepts is inadequate for effective teaching and learning as the justifications offered by most of the participants revealed deficits in their probabilistic and statistical reasoning. As stated, novice teachers need deep reasoning in statistics and probability information (Groth, 2017) to develop statistical and probabilistic reasoning in students by addressing misconceptions through games with critical questions (Hokor, 2020).

Teachers' mastery of statistical concepts has implications for their teaching strategies and assessments. The majority of the justifications provided suggest that novice teachers would be unable to develop probabilistic and statistical reasoning in their students. This is a call on teacher educators to provide real-life contexts in training teachers in probabilistic and statistical concepts (Hokor, 2023). Professional development programs should be designed on basic concepts of probability and statistics to shape novice teachers' reasoning in this area, especially the interpretation of probability and statistics information and various contexts of random events. This would enable them to implement the curriculum effectively and efficiently. The development of statistical literacy in students is premised not only on effective curriculum, but equally on creative and innovative teaching that promotes critical thinking and creates opportunities for students to decide based on fact and evidence, and not emotions. Teachers have a duty and responsibility to empower their students to ensure that objective facts do not appeal less in shaping the well-being of society than emotions and personal beliefs. Real-life contexts involving possible emotional situations should be created and students are made to exercise a judgment based on sound reasoning. This would enable them to be conscious of their emotions in the assessment of situations when deciding on uncertain situations. Several respondents failed to advance arguments that influence their choice of options in this study is an indication that they would have difficulty using critical questions to promote statistical and probabilistic reasoning. These respondents' answers seem to be anchored on emotions or personal beliefs as no reason or evidence was given. The ability to depart from emotions and what was unconsciously assumed is a difficult one, but it's an important issue statistics education should address for learners to exercise good judgment or make decisions in real-life contexts. With this, teachers can equip their students to become statistically literate citizens who can identify poorly presented information, fake news, and notice missing facts and challenge people's arguments and weighting of variables that lead to their conclusion on various public discourses.

Educators need to be aware of the potential for misconceptions and errors in thinking among learners when teaching probabilistic and statistical concepts. Teachers should provide ample opportunities for students to practice applying these concepts in various contexts and provide feedback to help students identify and correct errors in their reasoning. Additionally, teachers can use examples of correct reasoning provided by participants in Table 5 to help students develop a better understanding of probabilistic and statistical concepts. By highlighting correct reasoning, teachers can help students build a strong foundation in these concepts and avoid common errors and misconceptions. Teachers would become conscious of their misconceptions and engage in more professional development programs for deeper understanding.

Reasoning Patterns in Equiprobability bias Tasks

Anticipation of new teachers' ability to address equiprobability bias in basic students is problematic. They would have difficulty in targeting this misconception with activities and critical questions. After formal instruction on probability, students continue to demonstrate equiprobability bias (Hirsch & O'Donnell, 2001). However, research activities that directly target these misconceptions will be helpful (Sharma, 2016). The presence of equiprobability bias in teachers and its' implications goes beyond teaching to negatively affect their decision-making in real life. The overall goal of statistics education is to empower people to critically evaluate, analyze and make decisions that are relevant not only to themselves but also to the progress of society.

Many people tend to unconsciously believe that every situation that involves two or more possible outcomes is equally likely without analysis of each possibility. When respondents were asked which one is most likely when a spinner with ten equally divided sectors with four white parts and six red parts, some of them responded equally likely in this case. For instance, PR1922 "*Both white and red are all equally likely to land or occur since they are equally divided sectors*". This respondent considers only "equally divided sectors" without linking it with the number of each colored part. The response here shows the respondent's understanding of possibility but is not specific to this context. The respondent needed to compare the probabilities of the two cases. The equiprobability misconception continues to be a challenge to most people (Hokor et al., 2022). In this regard, many researchers would have to focus on how teachers design and implement activities aimed to address equiprobability misconceptions. This would provide clarity on the way teachers are teaching on its potential to help students to overcome their misconceptions.

Teachers' Representativeness Bias

We noticed representativeness bias in the pattern of novice teachers' reasoning responses. These misconceptions and errors highlight the need for better training and understanding of probability concepts among teachers. The study's findings have several implications for teaching and learning in statistics education. It highlights the need to address common misconceptions in probability and statistics among novice teachers. This could involve designing instructional materials that specifically target these misconceptions and providing opportunities for teachers to engage in professional development activities that focus on enhancing their understanding of probability concepts. Also, the study underscores the importance of promoting active learning strategies that engage students in problem-solving and critical-thinking activities. These approaches are effective in promoting deep learning and enhancing students' understanding of probability concepts (Hokor, 2020). Teachers could incorporate activities such as case studies, simulations, and inquiry-based learning to help students develop a better understanding of probability concepts in real-life contexts.

The importance of using real-world examples and contexts in teaching probability concepts has been emphasized. This approach can help students connect abstract concepts to real-life situations, making learning more meaningful and relevant. Teachers could incorporate examples from different fields such as games, business, and healthcare to illustrate probability concepts.

The findings here aligned with Paul and Hlanganipai (2014) who found representativeness bias was more stable in students. This implied that the misconceptions in high school were not addressed at tertiary institutions. Nevertheless, activities for professional development programs on probability concepts should pay more attention to teachers in addressing representativeness misconceptions. Also, teacher educators should provide appropriate learning experiences that are capable of eliminating these differences in both male and female teachers. It is important to provide probabilistic situations that relate to learners' daily lives. The daily experiences of both genders ought to be considered in task design and instructions.

The context provided must open for discussions on the law of small and large numbers. Many novice teachers hold the view that the toss of a coin as small as four should have two heads and two tails. This position is an indication of basic deficits in the law of small numbers. Similarly, most of the respondents expected all six possible different outcomes in the toss of an ordinary die six times. For example, in the roll of a die five times with the sequence 51246 when asked what would be the outcome on the next roll, then the respondent PR8622 responded; "*3 is the most likely to occur on the next roll because looking at the trend of the already occurred numbers, all the numbers except 3 has not yet occurred. So, it has a high probability or chance of occurring*." This suggests that the die keeps a record of the previous outcome and has the intention to produce a different outcome. In an in-service program or classroom, teachers and students should have the opportunity to roll the die and compare the results to the predicted outcome. This approach will address the basic deficits in reasoning and also provide clarity on the meaning of random in context.

In summary, the study underscores the importance of promoting gender equity in statistics education. The findings indicate that male and female novice teachers have reasoning patterns and misconceptions about probability. Teachers could be mindful of these patterns for gender-sensitive teaching and learning strategies that support all students learning needs with regard to culture. Teachers need to be aware of these misconceptions and find ways to correct them. For instance, teachers can use visual aids or real-life examples to help students understand probability better and avoid both equiprobability bias and representativeness bias. Additionally, teachers can provide opportunities for students to practice solving probability problems to help them avoid generalizations that are not supported by context and understand that each problem is unique.

CONCLUSION

This study investigated novice teachers' probabilistic and statistical reasoning in general and noted patterns of reasoning in their responses. The results of this study suggest that novice teachers who took about two statistics courses in their professional training continue to demonstrate high misconceptions. The males and females differ in general statistical and probabilistic reasoning in favor of males. Additionally, both reasoning mean scores were below the average score. Also, we found patterns of reasoning such as misconceptions in their ability and capacity to address equiprobability bias and representativeness bias. These patterns include misunderstanding of probabilistic and statistical contexts, and generalizing without adequate information. Therefore, professional development programs should focus on addressing novice teachers' misconceptions within context. This will in turn prepare both male and female teachers on their ability to develop statistical and probabilistic reasoning in primary and middle school learners.

Based on the findings, we call for further study that will investigate pre-service teachers' prior statistics and probability misconceptions compared to their misconceptions after college statistics courses. This will help assess the extent to which college statistics instructions develop probabilistic and statistical reasoning and to a large extent address misconceptions.

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APPENDIX

Probabilistic and Statistical Reasoning Assessment

This assessment is for research purpose to improve teaching and learning of probability and statistics. You are required to answer all questions honestly. Do and leave all workings or rough work on the paper.

Duration: 1 hour

Index number…………………………………………………Gender………………Age…..…

Programme………………………………………………………………….………………..

Religion ……………………………….. (E.g. Christianity, Islam, Judaism, Hinduism, etc.)

Read each statement carefully and **circle** the letter of the correct (best) option. In most cases, the next question that follow demand for **reason(s)** for your choice of the letter in the **space provided**.

1. If you rolled an ordinary die five times and obtained 51246, which of these is **most likely** to occur on your next roll?

a. 1
b. 2
c. 3
d. 4
e.5
f.6
g. The options a, b, c, d, e, and f are all equally likely.

1b). Give reason for your answer to question "1"
……………………………………………………………………………………………………
……………………………………………………………………………………………………
…………………………………………………………………………………………………

2. The following message is printed on a bottle of prescription medication:

WARNING: For applications to skin areas there is a 15% chance of developing a rash. If a rash develops, consult your physician. Which of the following is the best interpretation of this warning?

a. Don't use the medication on your skin — there's a good chance of developing a rash.
b. For application to the skin, apply only 15% of the recommended dose.
c. If a rash develops, it will probably involve only 15% of the skin.
d. About 15 of 100 people who use this medication develop a rash.
e. There is hardly a chance of getting a rash using this medication.

3. A spinner with ten equally divided sectors in which four parts are white and the rests are red is about to be spin, which color is the spinner **more likely** to land?

a. White
b. Red
c. Both white and red are all equally likely.

3b). Give reason for your answer to question three

……………………………………………………………………………………………
……………………………………………………………………………………………
………………………………………………………………………………

4. A small object was weighed on the same scale separately by nine students in a science class. The weights (in grams) recorded by each student are shown below.

6.2 6.0 6.0 15.3 6.1 6.3 6.2 6.15 6.2

The students want to determine as accurately as they can the actual weight of this object. Of the following methods, which would you recommend they use?

a. Use the most common number, which is 6.2.
b. Use the 6.15 since it is the most accurate weighing.
c. Add up the 9 numbers and divide by 9.
d. Throw out the 15.3, add up the other 8 numbers and divide by 8

5. If you toss a fair coin and get heads two times in a row, which of these is **most likely** on the next toss?

a. Tail
b. Head
c. Both head and tail are all equally likely.

5b). Give reason for your answer to question "5"

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6. Half of all newborns are girls and half are boys. Hospital A records an average of 50 births a day. Hospital B records an average of 10 births a day. On a particular day, which hospital is more likely to record 80% or more female births?

a. Hospital A (with 50 births a day)
b. Hospital B (with 10 births a day)
c. The two hospitals are equally likely to record such an event.

6b). Give reason(s) for answer to question "6".

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7. A fair die is to be toss six times, which of these is **most likely** to occur?
   a. 251634
   b. 123456
   c. 666341
   d. 132244
   e. The options a, b, c, and d are all equally likely

   7b). Give reason for your answer to question “7”

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8. A fair coin is to be toss four times in a row, which of these is **least likely** to occur?
   a. THTH
   b. TTTH
   c. HHHH
   d. The options a, b and c are all equally likely

   8b). Give reason for your answer to question “8”

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9. Two fair dice are to be tossed once. Among the options listed, which of them is the **least likely** to occur?
   a. Obtaining a sum of 11.
   b. Obtaining a sum of 10.
   c. A sum of 11, and a sum of 10 are all equally likely.

   9b). Give reason for your answer to question “9”

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10. If you toss a fair coin and get heads 3 times in a row, what is the chance of getting a head on the next toss?
   a. 1
   b. Greater than $\frac{1}{2}$
   c. $\frac{1}{2}$
   d. Less than $\frac{1}{2}$
   e. 0

   10b). Give reason for your answer to question “10”

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11. A die is numbered 1, 2, 3, 4, 4, 5. When the die is thrown once. Which of these is **most likely**?

a. 1
b. 2
c. 3
d. 4
e. 5
f. They are all equally likely

11b). Give reason for your for your to question “11”

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12. German MP is to toss two dice once. Among the options listed, which of them is **mostly likely** to occur?

a. Obtaining a sum of 4.
b. Obtaining a sum of 7
c. Obtaining a sum of 11
d. All of them have equal chance of occurring that is a, b, c.

12b). Give reason to your answer in question “12”

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13. A fair coin is to be toss four times in a row, which of these is **most likely** to occur?

a. THTH
b. TTTH
c. HHHH
d. The options a, b and c are all equally likely

13b). Give reason for your answer in “13”

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