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Racial Disparities in Advanced Placement (AP) Mathematics Exams: A Trend Analysis Using Participation Disparity Indices

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Abstract

Advanced academics such as Advanced Placement (AP) program, have served as a natural curriculum delivery option for motivated and talented high school students for decades. However, prior studies have indicated that students from certain racial groups have been historically unrepresented in these prestigious programs (Shores et al., 2019). Despite vast number of studies on racial disparities in these programs, research investigating the direction and magnitude of racial disparities is scarce. In this study, the authors analyzed data of over 10,000,000 students who participated in the Advanced Placement (AP) mathematics exams from 1997 to 2020 using trend analysis across races to understand the magnitude and direction of the excellence gaps. Our findings indicated that Native American, Black, and Hispanic students have been widely underrepresented in all AP mathematics exams across years; however, the trend analysis showed significant upward trends for Native American, Hispanic, Black students. Possible impacts of these findings within the context of the underrepresentation in STEM fields were also discussed.

Keywords: Racial disparities, Advanced Placement, High Achievement, Excellence Gaps, Participation Disparities, Advanced Mathematics, Advanced Academics

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INTRODUCTION

Over several decades, advanced programs such as Advanced Placement (AP), International Baccalaureate (IB), and dual enrollment have served as a natural curriculum delivery option for motivated and talented high school students (Hertberg-Davis & Callahan, 2006, 2008; Robinson et al., 2007). These programs have provided participants with access to advanced content while having them accelerated (Kettler & Hurst, 2017). Besides meeting the academic needs of talented students, participation and achievement in these advanced programs has been a major predictor for future career decisions too. Research shows that enrollment in advanced high school courses leads to a greater chance at college acceptance (Ackerman et al., 2013; Adelman, 1999), a higher college GPA (Hargrove et al., 2008), higher college retention and graduation rates (Lowe, 2016), and higher academic confidence and motivation to pursue career aspirations (VanTassel-Baska, 2000).

Despite their significant academic and career benefits, many studies and institutional reports have repeatedly pointed to significant racial disparities in these rewarding programs (Bahar, 2022; Bahar et al., 2022). Research in mathematics education helps to address the impact of race issues in mathematics and mathematics education (Martin, 2009, 2013), Black-White achievement gaps in mathematics (Lubienski, 2002), race-gender identities in mathematics (Leyva et al, 2021) and racial groups' access to advanced mathematics coursework (Battey, 2013). These works highlight racial achievement gaps and related factors, but do not adequately examine excellence gaps in AP mathematics (Plucker & Peters, 2016) nor students with gifts and talents in mathematics toward expected levels eminence (Subotnik et al., 2011). Researchers have identified that students from rural areas and low-income families, females, and Black, Hispanic, and Native American students were historically underrepresented among advanced academics participants (Hertberg-Davis et al., 2006; Kettler & Hurst, 2017; Prong, 2018; Wanzo, 2014). According to a recent report (Center for Education and Civil Rights, 2021), Hispanic and Black secondary school students were up to four times less likely to enroll in AP courses in Virginia public schools, compared to their share in school enrollment. In another study, the female to male ratio among AP Calculus BC exam takers was found to be roughly 0.7, which indicated that females were substantially underrepresented in the AP exam rooms (Bahar, 2021b).

The literature on disparities in advanced programs includes a substantial number of studies (Bahar, 2021a; Gagnon & Matttingly, 2016; Naff et al., 2021; Shores et al., 2019). However, research investigating the direction and magnitude of racial disparities is scarce. For instance, through a comprehensive analysis of AP participation and achievement across various U.S. school districts, Gagnon and Mattingly (2016) emphasized the need for investigations revealing the uneven rates of AP participation and success among disadvantaged student communities. They stressed that a deficiency in opportunities to take advanced courses would pose a significant risk to equality and social justice. In a similar vein, linking disproportionality across multiple educational outcomes, Shores and his colleagues

(2019) explored categorical inequality in Advanced Placement course-taking across student groups. The researchers drew attention to the urgency of quantifying these disparities to an extent that these differences were largely the result of discretionary practices. Addressing these calls to inform the literature, in this study, we focused on racial disparities in AP mathematics exams and analyzed the trends in participation across races to explore the magnitude and direction of gaps. Developing new indices free from white-centeredness, we documented how to monitor disparities over the years across students from diverse racial groups by using robust techniques.

Status Quo of Disparities in Advanced Academics

Addressing concerns over the disparity issues in advanced programs, large-scale policies have been initiated by federal and state agencies and many funding/grant programs were made available for the use of school districts to expand access to advanced programs. For example, recently, Congress allocated $1.2 billion to the Title IV, Part A Student Support and Academic Enrichment Grants program, which would provide funds to cover a portion or the whole cost of Advanced Placement Exam fees for low-income students in all schools as of 2020 (College Board, 2021). Like efforts at federal level, many states adapted new policies and mandates for expanding the access for advanced programs, which includes but not limited to requiring all high schools/districts to offer AP or IB programs, funding for teacher trainings, state subsidies for testing fees for advanced programs (Education Commission of the States, 2021). For example, in 2016, The Florida Partnership for Minority and Underrepresented Student Achievement program started to offer AP teacher training to those specific, targeted districts and schools to increase the number of teachers that can teach AP classes. In a similar vein, the Advanced Placement Incentives Pilot Program in Colorado was formed in 2014 with the goal of increasing access to AP courses in remote schools while also increasing participation in AP programs by students who are enrolled in the school lunch program (Education Commission of the States, 2021).

Despite these concerted federal and state efforts, inequities in participation and achievement in advanced programs persist for certain populations. In particular, racial disparities are immensely concerning as recent trends indicate that current efforts and initiatives have fallen short to surmount the gaps across races in the long run (Center for Education and Civil Rights, 2021). Given the reputation of the advanced academics in predicting future educational attainments and career success (Breyer, 2021), reducing racial disparities to none in these programs is crucial because they are not only a major educational issue but also a pertinent social justice problem (Bahar & Maker, 2020; Ford et al., 2008; Graefe and Ritchotte, 2019).

AP Mathematics Program

The Advanced Placement (AP) program was founded in the mid-1950s by the College Board, a nonprofit education organization, with a goal to give college-level instruction to high-achieving high school students (Christiansen, 2009). The program started with a total of 12 Advanced Placement (AP) courses, including AP Calculus, and it reached over 1,000 students across 110 schools in its first year (Demaree, 2016). At the end of 2019, more than 5 million AP tests were taken by 2.8 million students at 20,000 high schools throughout the United States (College Board, 2020a). According to College Board, over 90 percent of high schools in the United States offered at least one AP course in 2020 (College Board, 2020c).

As of this writing, the AP program provides 38 distinct courses in 22 different topics, ranging from science to art (College Board, 2019). The AP program offers three mathematics courses: AP Calculus AB, AP Calculus BC, and AP Statistics. According to College Board, over 500,000 mathematics exams were taken in one of these three subjects in the 2020 examination year. AP Calculus AB is the most popular mathematics course in the program. As of 2020, over 250,000 students took the AP Calculus AB exam. The content of the Calculus AB course is roughly similar to that of a first-semester college calculus course, covering differential and integral calculus concepts (College Board, 2020a), and students often take it after Precalculus. According to College Board National Reports, fewer than 20% of the participants achieved a score of 5. Despite many similarities in the content, AP Calculus BC course covers more advanced techniques and knowledge about calculus topics compared to Calculus AB. In the 2020 examination, roughly 115,000 students took the AP Calculus BC exam, with approximately 40% of participants scoring a 5. The AP Statistics program is the most recent addition to the AP program's math courses. In 1997, the program's first test was held. The number of students taking the AP Statistics test rose faster than the number of students taking the AP Calculus AB and BC examinations. More than 175,000 pupils sat for the test in 2020, with over 15% receiving a perfect score.

Almost all AP courses are year-long courses followed by end-of-year exams, which are administered by the College Board in a standardized format to assess the content and skill mastery of the course (College Board, 2020a). AP exams are criterion-referenced tests, with students' exam scores reported on a 5-point scale, with 5 representing the highest possible score (Shaw et al., 2013). College Board guidelines recommend that a score of 5 is a college-grade equivalent of "A," while a score of four is an "A−, B+, or B," and a score of three equates to a "B−, C+, or C." (Bahar, 2021b). In 2019, students reported their exam scores to over 3,500 colleges to seek course credit. Most colleges and universities provide credit for AP scores of three or above, however this varies from institution to institution (College Board, 2019).

Present Study

This study would contribute to literature in several ways. First, we aimed to investigate the trends in racial disparities in AP mathematics examination. Given that the disparities emerging during K-12 years can be more detrimental because of their ubiquitous and predictive effects, monitoring and examining these disparities through scientifically sound techniques is crucial (Berends, 2005). Examining and understanding the disparities in programs, which have been targets of wide critiques due to longstanding racial inequities – such as Advanced Placement programs – would be informative for taking actions to sustain access and equity in educational opportunities for all. Despite the importance of the topic, to our knowledge, no prior studies examined the trends in racial disparities in AP mathematics exam. In this study we will employ trend analysis to examine the disparities. Due to its power to highlight the patterns of the data, use of trend analysis on this matter would provide future researchers with evidence to investigate the direction and magnitude of the disparities over years (Bahar, 2021b).

Second, literature involves a substantial number of studies and reports that have investigated racial disparities in educational settings; however, only a handful of them used indices to compare disparities across diverse groups. Use of indices during examination of disparities allows researchers a systematic procedure to quantify the inequities among groups. In this study, the authors created new indices, using a diverse method. In almost all of the studies that benefitted from indices to detect racial disparities, the authors chose to use baseline groups, which is generally White population. For example, the achievement rates of Black participants are compared with those of their White peers. Different from these studies, we chose to compare each racial groups' participation and achievement to their own representation in secondary school enrollment and AP exam participation respectively. We think such a perspective is more robust, given that the patterns of participation in White population evolve over time too. With the speedy evolution of ethnic and racial compositions in the U.S., comparing each racial group to their overall representation would be a more accurate procedure. Moreover, comparing each group to their representation in overall population helps to "decenter whiteness as the standard form" (Corporation for Supportive Housing, 2021). Combining all these arguments and addressing the needs stated, the findings of this study will significantly inform the literature as the methodology and indices we introduced could be modeled by further studies to monitor racial disparities in other domains. The following research question guided this study: What are the trends in racial disparity among participants in AP math exams?

METHOD

Participants

For this study, we analyzed the test results of roughly 10,000,000 students who took the AP mathematics examinations from 1997 to 2020. The data regarding the AP exam participation and top achievement came from the College Board's AP National Summary reports, which are issued once a year. These reports include specific information on AP Test participation, volume, and performance, organized by exam type, grade level, race, and gender (College Board, 2020c). We only used the data from three mathematics examinations (Calculus AB, Calculus BC, and Statistics) for the analyses. Table 1 depicted the number of participants in each of these three exams from 1997 to 2020.

Table 1. Number of participants in AP mathematics exams across years

Year	AP Calculus AB					AP Calculus BC					AP Statistics				
	Native	Black	Hispanic	Asian	White	Native	Black	Hispanic	Asian	White	Native	Black	Hispanic	Asian	White
1997	363	4019	5144	16183	73219	36	394	630	6027	13032	26	313	390	1334	4849
1998	391	4336	5630	16859	76767	46	458	772	6686	15953	56	586	819	2486	9831
1999	464	5000	6529	18362	83463	69	550	944	7559	18381	80	845	1223	3999	16463
2000	495	5480	7617	20280	92848	97	614	1136	8506	21359	136	1244	1723	5822	22856
2001	501	5724	8697	21110	99378	78	678	1349	9539	23700	140	1535	2291	6914	27786
2002	568	6171	9653	23139	105922	96	783	1484	10077	26306	202	1950	2879	8248	33368
2003	628	6678	11240	23723	112352	126	952	1887	11281	28597	181	2263	3484	9317	39241
2004	670	6930	12184	25111	116704	137	1024	2232	12127	31069	245	2641	4293	10416	43946
2005	746	7842	13799	27134	121410	139	1125	2529	13697	32764	267	3283	5130	12184	50516
2006	756	8453	15040	29049	124190	171	1227	2644	14721	34191	325	3940	6016	14142	55841
2007	823	9329	17329	31922	133973	154	1404	3196	16794	37363	373	4596	6928	15410	63320
2008	805	10290	19830	33147	139801	187	1571	3718	18066	39604	405	5603	8705	17480	68700
2009	875	11325	21711	35350	141748	216	1701	4202	19292	40819	452	6168	9829	19001	72749
2010	913	12205	24053	37608	146283	233	1865	4647	20571	42789	487	7046	11208	21017	78712
2011	1026	13467	27215	40125	151252	235	1925	5432	22709	45027	553	7955	13379	23575	85982
2012	1095	13852	29506	42520	154407	246	2109	6007	24794	48094	606	8909	14823	25337	90398
2013	1287	14908	34726	46536	162844	354	2649	7325	27875	53059	746	9997	17807	29016	99637
2014	1299	15974	38055	49164	166216	333	3068	8416	29942	55999	844	10935	21117	31741	105963
2015	1291	16862	40815	50917	166333	338	3181	9303	31200	58589	842	11290	23597	33576	110106
2016	1172	15379	49747	49419	164060	293	2969	12042	31893	59497	837	10861	30238	33771	112431
2017	1210	15443	53547	51577	163561	330	3142	13148	34291	61694	841	11283	33100	35959	114487
2018	1115	14535	51445	51570	157467	288	3317	14259	36547	62776	789	11005	33428	38750	115606
2019	1045	14037	51365	51228	151329	281	3179	13976	38087	61508	730	10758	33579	39945	111783
2020	958	10880	41748	47554	134440	315	2826	11929	35566	54920	732	7116	26680	35192	96418

Testing Procedures and Setting

In terms of testing structure and procedures, the three AP mathematics examinations, Calculus AB, Calculus BC, and Statistics, are comparable. The exams, as are many other AP exams, are proctored on set days, often in the first two weeks of May at thousands of high schools and approved testing centers around the country (Bahar, 2021b). Even though students have the option of choosing their testing place, most students prefer to take these tests at their school during regular school hours. The tests are submitted back to the College Board facilities to be graded, after certified school staff have proctored them (Bahar, 2021b). A multiple-choice portion with 40 to 45 multiple-choice questions, and a free-response section with six free-response questions, are included in the tests. In each exam, there are some parts that allow students to use a graphing calculator (College Board, 2020a). The tests roughly last three hours and fifteen minutes, with each section having equal weight.

AP exam scores, like many other criterion-referenced examinations, are determined based on mastery of the skills and knowledge on the subject rather than on a curve (Sundquist, 2016). Because of their differing formats, multiple-choice and free-response sections are evaluated separately. The free-response sections are graded by authorized educators, including experienced high school AP instructors and college professors at summer conventions, while the multiple-choice parts are scored by computers. To maintain consistency over years, the raw scores from each section are summed to generate a composite score. Using statistical procedures, the composite scores are then converted into a five-point scale, one being the lowest and five being the highest possible score (Bahar, 2021b).

Inter-rater reliability statistics for different sections of the Calculus AB and BC examinations ranged from high 0.70s to low 0.90s (Bridgeman et al., 1996). Furthermore, according to research that investigated the predictive validity of AP mathematics exams, there is a significant positive relationship between AP exam achievement and college course placement (Mattern et al., 2009; Patterson and Ewing, 2013).

Data Analysis

Disparity Index Calculations

Researchers have used various indices to examine disparities in academic participation and performance (Bahar; 2021a, 2021b; Bahar et al., 2023; Ellison and Swanson, 2010; Hyde et al., 1990; Hyde et al., 2008; Lindberg et al., 2010; Olszewski-Kubilius and Lee, 2011; Robinson and Lubienski, 2011). These indices were designed to monitor the gender disparities among top and low performing students in mathematics achievement. Ford (2013) provided guidance for the field of gifted education to quantify under-representation using a racial equity index, and Gentry et al. (2019) used a representation index to examine "missingness" of racial groups in gifted and talented education programs. However, to our knowledge, no indices were identified in prior studies to analyze the racial disparities in mathematics achievement.

Participation Disparity Index (PDI). In this study, to measure and monitor the racial disparities in participation in AP mathematics exams across years, the authors created a participation disparity index (PDI). The PDI depicts the representation of students from a specific racial group in each AP exam, compared to their representation in the overall student enrollment in the U.S. secondary schools. The PDI index is exam specific and calculated as:

$$PDI = \frac{\%\ of\ exam\ participants\ (PEP)\ from\ a\ specific\ racial\ group}{\%\ of\ enrollment\ in\ the\ US\ secondary\ schools\ for\ the\ corresponding\ race\ (PECR)}$$

As seen in the formula above, the PDI index is a ratio of two separate indices, PEP and PECR. The PEP index was determined as the representation of each racial group in an exam participation. To calculate PEP, the data regarding the percentage distribution of AP exam participation across races were obtained from College Board's database. After downloading the data from the College Board National summary reports from 1997 to 2020, the PEP index was calculated across years. Likewise, the PEP is exam specific and calculated for each racial group as:

$$PEP = \frac{\#\ of\ exam\ participants\ from\ a\ specific\ racial\ group}{\#\ of\ total\ exam\ participants}$$

The PECR index was determined as the representation of each racial group in the enrollment in the U.S. secondary schools. The PECR values were obtained from the National Center for Education Statistics (NCES) database. As the primary federal institution, the NCES collects data and publishes reports related to education in the United States (Institute of Education Sciences, 2021).

The PDI index basically identifies whether there is parity in the representation of students from a racial group in an AP exam compared to their enrollment. For example, let's assume the percentage of White enrollment in secondary schools is 50 percent (PECR). When the percentage of White students who participated in the AP Calculus AB exam is equal to 50 percent, then the value of the PDI function should be equal to 1, which indicates a parity for the White students for the corresponding exam. When the percentage of White students who participated in the AP Calculus AB exam is less than 50 percent, then the value of the PDI function becomes less than 1, and an underrepresentation occurs for the Whites. Likewise, when the percentage of White students who participated in the AP Calculus AB exam exceeds 50 percent, the value of the PDI gets larger than 1, which indicates an overrepresentation for White students for the corresponding exam.

Analysis of Research Question

First, AP participation and top achievement data from the College Board's AP National Summary reports were obtained from 1997 to 2020, and the secondary school enrollment data from the National Center for Education Statistics (NCES) database (NCES, 2021). Second, the participation disparity index (PDI) was calculated for all three of AP exams by races across years. The calculated PDIs were later employed in the data analysis to answer the research question.

To examine trends in gender disparities in participation on AP mathematics examinations across races we employed a Mann-Kendall (MK) trend test. The MK trend test is a strong exploratory analytic tool that has been widely used to detect the presence of monotonic patterns in time-series data (Mann, 1945; Kendall, 1975). According to Jaiswal et al. (2015), the MK test is routinely used to examine trends in hydrological and climatic data; however, the test is not typically used to evaluate trends within education data sets (Bahar, 2021a). As a non-parametric test, MK does not require assumption of normality, while it is still effective with even small data sets. Nevertheless, the MK test requires three assumptions: (1) the data are distributed independently, (2) the measurements truly represent states of the observations, and (3) an unbiased use of methods during sample collection and instrumental measurements (Hirsch et al., 1982).

Confirming that these three assumptions were not violated, using XLSTAT software, the MK trend analysis together with Sen’s non-parametric procedures were performed. Sen’s procedures included the calculation of the magnitude of the slope of the trend lines over time which were not provided by the MK analysis. Sen’s non-parametric procedures

provide a reliable assessment of the size of a trend and have been commonly used to determine the slope of trend lines (Sen, 1968; Yu et al., 2002).

While employing the MK tests, the disparity indices (let’s use the participation disparity index (PDI) as an example) were employed as time-series data, which were collected over time and were denoted as $\{PDI_{1997}, PDI_{1998}, \ldots, PDI_{2020}\}$. As the first step of the analysis, an individual sign function is calculated, as follow:

$$sgn(PDI_j - PDI_i) = \begin{cases} 1, & PDI_j - PDI_i > 0 \\ 0, & PDI_j - PDI_i = 0 \\ -1, & PDI_j - PDI_i < 0 \end{cases}$$

This function simply shows if the differences between the data from subsequent years are positive, negative, or zero, depending on the input data (Bahar, 2021a). In the next step, a MK test statistic, S is calculated:

$$S = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} sgn(PDI_j - PDI_i).$$

Having a positive S value implies an upward trend, while a negative S value suggests a downward trend. To assess the significance of the S value, the mean and variance of the S are calculated, followed by the Z-score. Lastly, the significance of the trends is determined by testing the null and alternative hypotheses (Bahar, 2021a).

RESULTS

Trends in Racial Disparity in Participation in AP mathematics exams

AP Calculus AB Exam

The number of participants in AP mathematics exams from 1997 to 2020 and participation disparity index (PDI) values, separated by race, were depicted in Table 1 and 2 respectively. Despite the rapid increase in the number of participants for all races, the PDI indexes showed that Native American, Black, and Hispanic students have been widely underrepresented in the AP Calculus AB exam rooms over the years (Table 2). According to the data in Table 3, the average PDI index values were 0.373 for Native Americans, 0.303 for Blacks, 0.482 for Hispanic, 3.493 for Asians, and 1.212 for Whites.

According to the Mann-Kendall trend analyses, the PDI values showed significant upward trends only for Native American, Hispanic, and Black participants over the years ($p < .001$ for all three races). Sen's slope analysis indicated that the largest increase in the PDI values belonged to the Hispanic students, which was roughly twice of those for Blacks and triple of Native Americans. The PDI trend for the Asian students was downward and statistically significant ($p < .001$) as the PDI values dropped from 4.11 to 3.51 over the years. Compared to their peers from other races, there is no trend for White students ($p < .001$) as their PDI values ranged from 1.14 to 1.21 between 1997 and 2020 (Table 2).

Table 2. AP Mathematics participation disparity ındex (PDI) across years

Year	AP Calculus AB					AP Calculus BC					AP Statistics				
	Native	Black	Hispanic	Asian	White	Native	Black	Hispanic	Asian	White	Native	Black	Hispanic	Asian	White
1997	0.30	0.23	0.35	4.11	1.14	0.14	0.10	0.20	6.99	0.93	0.24	0.21	0.30	3.84	0.86
1998	0.34	0.24	0.35	4.08	1.15	0.16	0.10	0.20	6.62	0.98	0.32	0.22	0.35	4.04	0.99
1999	0.33	0.25	0.36	3.96	1.16	0.19	0.11	0.21	6.41	1.00	0.27	0.20	0.32	4.06	1.08
2000	0.32	0.25	0.36	3.84	1.18	0.24	0.11	0.21	6.15	1.04	0.34	0.21	0.31	4.20	1.11
2001	0.30	0.24	0.37	3.66	1.20	0.17	0.11	0.21	6.08	1.05	0.29	0.22	0.33	4.05	1.13
2002	0.32	0.24	0.37	3.65	1.21	0.20	0.11	0.20	5.75	1.09	0.35	0.23	0.33	3.94	1.15
2003	0.33	0.25	0.39	3.44	1.22	0.23	0.12	0.23	5.72	1.09	0.27	0.23	0.33	3.75	1.18
2004	0.34	0.24	0.39	3.41	1.23	0.23	0.12	0.24	5.55	1.10	0.32	0.24	0.35	3.64	1.19
2005	0.36	0.26	0.40	3.41	1.23	0.22	0.13	0.24	5.70	1.10	0.30	0.26	0.35	3.61	1.21
2006	0.35	0.28	0.41	3.44	1.22	0.26	0.13	0.23	5.70	1.10	0.33	0.28	0.36	3.66	1.20
2007	0.35	0.28	0.42	3.40	1.23	0.21	0.14	0.25	5.74	1.10	0.34	0.29	0.35	3.47	1.23
2008	0.33	0.29	0.45	3.22	1.24	0.24	0.14	0.27	5.55	1.11	0.33	0.32	0.39	3.40	1.22
2009	0.34	0.32	0.46	3.25	1.23	0.26	0.15	0.28	5.54	1.11	0.34	0.33	0.40	3.38	1.22
2010	0.37	0.34	0.47	3.44	1.25	0.29	0.16	0.28	5.82	1.13	0.37	0.37	0.40	3.56	1.25
2011	0.40	0.36	0.49	3.35	1.24	0.28	0.16	0.30	5.76	1.13	0.38	0.38	0.42	3.46	1.25
2012	0.41	0.36	0.50	3.36	1.24	0.27	0.16	0.30	5.73	1.13	0.39	0.40	0.43	3.43	1.25
2013	0.49	0.36	0.53	3.41	1.23	0.38	0.18	0.32	5.75	1.13	0.47	0.40	0.45	3.50	1.24
2014	0.48	0.38	0.55	3.47	1.23	0.33	0.20	0.33	5.77	1.13	0.49	0.41	0.48	3.54	1.24
2015	0.46	0.39	0.57	3.39	1.22	0.32	0.20	0.34	5.52	1.15	0.46	0.40	0.50	3.43	1.24
2016	0.42	0.36	0.67	3.19	1.21	0.27	0.18	0.42	5.33	1.14	0.44	0.37	0.60	3.23	1.23
2017	0.42	0.35	0.70	3.21	1.20	0.29	0.18	0.43	5.34	1.13	0.43	0.37	0.63	3.25	1.22
2018	0.40	0.34	0.68	3.31	1.20	0.24	0.18	0.44	5.47	1.12	0.39	0.36	0.61	3.43	1.22
2019	0.39	0.34	0.69	3.32	1.20	0.24	0.18	0.43	5.61	1.11	0.37	0.36	0.62	3.52	1.21
2020	0.40	0.30	0.64	3.51	1.21	0.29	0.17	0.40	5.80	1.10	0.44	0.28	0.58	3.67	1.23

Table 3. Mann-Kendal Trend Analysis of Participation Disparity Index (PDI) across Years

AP Exam	Variable	Trend Test Values								Sen's Slope Values					
		Min	Max	M	SD	K's tau	S	Var(S)	p*	Slope	SLB	SUB	IV	ILB	IUB
Calculus AB	Native	0.300	0.491	0.373	0.054	0.638	176.00	1625.333	<0.001**	0.005	0.004	0.007	-10.112	-12.112	-8.621
	Black	0.234	0.394	0.303	0.054	0.652	180.00	1625.333	<0.001**	0.007	0.005	0.009	-12.944	-14.935	-11.229
	Hispanic	0.354	0.698	0.482	0.119	0.942	260.00	1625.333	<0.001**	0.015	0.012	0.017	-29.460	-31.596	-26.761
	Asian	3.189	4.111	3.493	0.262	-0.536	-148.00	1625.333	<0.001**	-0.024	-0.039	-0.009	51.717	36.931	67.029
	White	1.142	1.251	1.212	0.029	0.239	66.00	1625.333	0.108	0.002	0.000	0.004	-2.346	-4.474	-0.157
Calculus BC	Native	0.136	0.380	0.249	0.056	0.587	162.00	1625.333	<0.001**	0.006	0.003	0.009	-11.570	-14.557	-8.923
	Black	0.103	0.198	0.146	0.032	0.790	218.00	1625.333	<0.001**	0.004	0.004	0.005	-8.443	-9.070	-7.883
	Hispanic	0.198	0.441	0.290	0.082	0.920	254.00	1625.333	<0.001**	0.010	0.008	0.012	-19.696	-21.685	-18.087
	Asian	5.334	6.988	5.808	0.392	-0.457	-126.00	1625.333	0.001**	-0.030	-0.053	-0.013	66.793	49.394	89.321
	White	0.928	1.145	1.091	0.054	0.652	180.00	1625.333	<0.001**	0.005	0.003	0.007	-8.298	-10.365	-6.678
Statistics	Native	0.243	0.489	0.360	0.067	0.638	176.00	1625.333	<0.001**	0.008	0.005	0.010	-16.052	-17.969	-13.164
	Black	0.200	0.409	0.306	0.074	0.674	186.00	1625.333	<0.001**	0.010	0.007	0.012	-19.869	-22.181	-17.143
	Hispanic	0.304	0.627	0.426	0.109	0.899	248.00	1625.333	<0.001**	0.013	0.010	0.016	-24.742	-28.116	-22.057
	Asian	3.229	4.204	3.628	0.268	-0.536	-148.00	1625.333	<0.001**	-0.029	-0.040	-0.017	61.664	49.545	72.978
	White	0.857	1.247	1.180	0.093	0.587	162.00	1625.333	<0.001**	0.006	0.004	0.010	-11.767	-15.728	-8.867

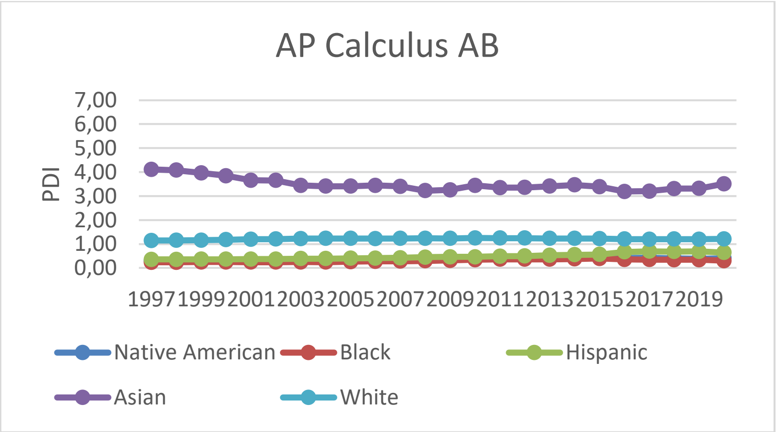
Note. * p<.05, ** p<.01.

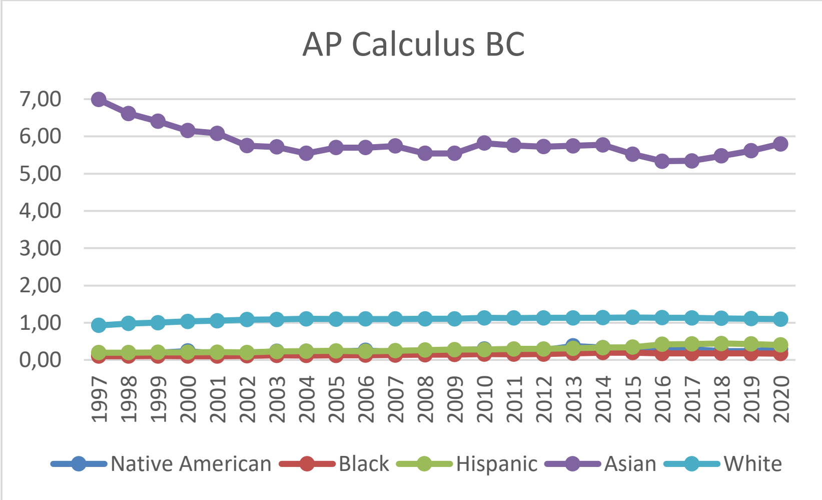
AP Calculus BC Exam

Similar to the AP Calculus AB exam, the number of AP Calculus BC exam participants increased rapidly from 1997 to 2020 for all races (Table 1). Likewise, the PDI indexes showed that Native American, Black, and Hispanic students have been widely underrepresented among the exam participants (Table 2). According to the data in Table 3, the average PDI index values were 0.249 for Native Americans, 0.146 for Blacks, 0.290 for Hispanics, 5.808 for Asians, and 1.091 for Whites. According to the Mann-Kendall trend analyses, the PDI values showed significant upward trends for Native American, Hispanic, Black, and White participants over the years ($p < .001$ for all four races). Sen's slope analysis indicated that the largest increase in the PDI values belonged to the Hispanic students, while the smallest growth was for Blacks. The PDI trend for the Asian students was downward and statistically significant ($p = .001$) as the PDI values dropped from 6.99 to 5.80 over the years (Table 2).

AP Statistics Exam

Like the other two exams, the number of students taking the AP Statistics exam increased significantly from 1997 to 2020 for all races. Similarly, according to the PDI index data, Native American, Black, and Hispanic students have been widely underrepresented among the exam participants (Table 2). According to the data in Table 3, the average PDI index values were 0.360 for Native Americans, 0.306 for Blacks, 0.426 for Hispanics, 3.628 for Asians, and 1.180 for Whites. The PDI index values for the Statistics exam were very similar to those of Calculus AB. The Mann-Kendall trend analyses indicated that the PDI values showed significant upward trends for all races over the years except Asians ($p < .001$ for all four races). Different from other groups, the PDI trend for the Asian students was downward and statistically significant ($p < .001$) as the PDI values dropped from 3.84 to 3.67 over the years (Table 2). Likewise, Sen's slope analysis indicated that the largest increase in the PDI values belonged to the Hispanic students, while the smallest growth was for Whites.





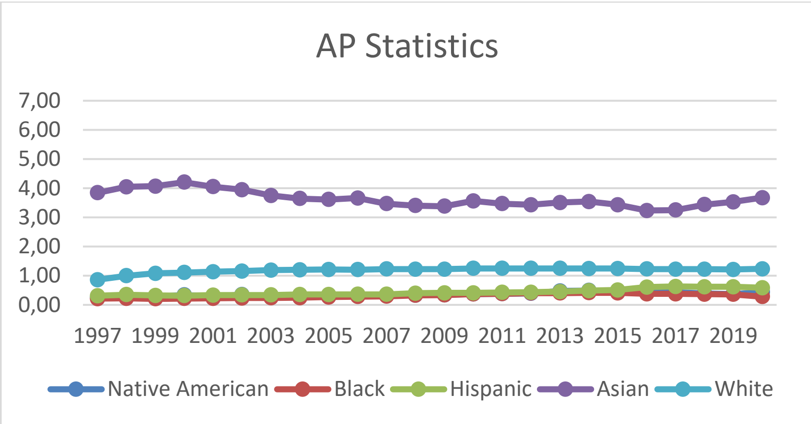


Figure 1. Trends in participation in AP mathematics exams

Note. FMR-TA = female-to-male ratio in top achievement; AP = advanced placement; Dashed lines (Linear) represent Sen's slopes.

DISCUSSION

From serving a thousand students in the mid-1950s, the Advanced Placement (AP) program achieved to be a premier secondary school program serving millions of students as of 2020. The participation data depicted in Table 1 validates the fact that the number of students who took the AP mathematics exams increased dramatically over the years for all students; however, the magnitude of the increase varied across races. For example, from 1997 to 2020, the number of Hispanic students who took the AP Calculus AB exam increased from 5,144 to 41,748, while the number of Black participants only grew to 10,880 from 4,109 during the same period. Given that the increases in the participation of racial groups in a given AP exam might have been affected by external variants such as increase of Hispanic students in their overall school enrollment, a robust measure of parity index that considers and controls covariant effect was needed. The indexes we created and used in this study can be used for this purpose, to monitor the racial disparity trends in participation in programs.

One significant finding of the study was that Native American, Black, and Hispanic students have been widely underrepresented in all AP mathematics exams across years. In addition, the PDI values indicated that Asian students have been dramatically overrepresented, while Whites have been around the equity line (See mean PDI values in Table 3). For example, the average Black representation, compared to their enrollment in the US secondary schools, were roughly .30, .14, and .31 on AP Calculus AB, Calculus BC, and Statistics exams respectively. These PDI values indicated that Black learners were seven times underrepresented in AP Calculus BC exam rooms compared to their overall school enrollment. The average PDI values on these exams were 3.50, 5.80, and 3.63 for Asian students. When combined, these two findings show that Asian students were 40 times overrepresented in AP Calculus BC exam rooms compared to Black students.

Another major finding of the study was that the Participation Disparity Index (PDI) values moved toward the parity line over the years for all races. According to the trend analysis, The PDI values showed significant upward trends for Native American, Hispanic, Black, and White participants over the years, while the trend was significantly downward for Asian American students. Given that the Asian American students were the only overrepresented group, these results imply that all racial groups have been moving toward the parity line, which might be accepted as an encouraging finding. Further, the trend behaviors regarding participation were consistent across three mathematics exams. One can infer from these promising trends that the policies and practices may have been designed to diminish the racial disparities in participation. Although such an inference holds true, Sen's slope analyses indicated that there is no imminent parity coming soon for any underrepresented groups. For example, according to Sen's slope results that the largest slope (change) in the PDI values belonged to the Hispanic students, they were closer to the equity line with the fastest growth trend compared to Black and Native American students across all three AP math exams. According to the slope analysis, even for the Hispanic students, reaching the parity line will take around three to four decades for AP Calculus AB and Statistics exams and almost six decades for the Calculus BC exam. What is worse, the period to close the gaps for Black and Native American students might take up to a century. Such a long parity period is unacceptable and without making investments in policy changes that impact substantive changes in practices. Prior studies have pointed to the fact that advanced academic programs, such as AP, may not be offered at schools that were heavily populated by students from underrepresented groups. Although this argument might be still true, given that over 90 percent of high schools in the United States have offered AP courses in 2020 (College Board, 2020c), there are obviously other factors preventing students, who are from underrepresented populations, from participating in advanced academics programs. As stated earlier Black students (and others from underrepresented populations) might possibly face implicit or explicit entry barriers to AP courses including subjective and biased gatekeeping measures (Chatterji, 2021), and race is a factor. The role of district level administrators in changing policy to provide support for pre-AP and other enrichment opportunities as early as middle school have not been clearly articulated.

In efforts to address the preparation of secondary school students for advanced instruction, early and sustained access to enrichment programs and 'front loading' are recommended (Plucker & Peters, 2016).

Another interesting finding was that despite the similarities in trend enrollments across three AP math exams, the disparities were wider on AP Calculus BC exam than those were for AP Calculus AB and AP Statistics. For example, the mean PDI for Asian students on Calculus BC exams between 1997 to 2020 was 5.81, while it was 3.49 and 3.63 for Calculus AB and Statistics exams. Similarly, the mean PDI for Black students on Calculus BC exams during the same period was 0.15, while it was 0.30 and 0.31 for Calculus AB and Statistics exams respectively. One possible explanation for the wider disparities on Calculus BC exam than for those on Calculus AB and Statistics might be related to the fact that most students consider participating in Calculus BC after a successful completion of Calculus AB or Statistics courses. Simply put, this means that the Calculus BC exam might be a platform where the disparities on earlier levels of mathematics participation, such as Calculus AB and Statistics, are exacerbated. This explanation is in line with prior studies, (Finkelstein et al., 2012; Lubienski et al., 2004) which found that the decision to enroll in higher level math courses usually was determined by earlier level of mathematics that students take. From a different point of view, some can suggest that larger disparities in higher levels of advanced programs might result from smaller disparities in earlier educational attainments (Bahar & Adiguzel, 2016). Regardless, we suggest that policies and practices that reduce the impact of systemic racism will in turn reduce disparities in participation or enrollment in earlier math courses would also help later disparities in participation in higher levels of mathematics.

Lastly, our findings showed that White learners were roughly on the equity line of the representation for all three AP mathematics exams. To our knowledge, earlier literature did not mention such information before. Given that White students were perceived as highly overrepresented in many educational opportunities and attainments, this finding might be thought-provoking. One possible reason why earlier studies consistently portrayed White students as overrepresented might be due to the way indexes were calculated to detect racial disparities. In many earlier studies, researchers used Whites as a benchmark group since they constitute the largest student population. Although the use of such methodology is helpful to compare disparities across racial groups, it evidently fails to compare each racial groups' participation and achievement to their own representation in secondary school enrollment and AP exam participation respectively.

Implications for Research

Research is needed to disaggregate the AP Math data among underrepresented students for school district leaders, principals, gifted education coordinators, and AP coordinators and teachers to understand their pattern of AP Math engagement at higher levels, particularly in the top achievers in the current era of Covid. One would expect that for the top achievers, underrepresented students would be equipped to sustain any enrollment gains that were found in this study. However, an analysis of this data is necessary to understand the impact of Covid. Are students from underrepresented groups more susceptible to lower performance, AP Math course drop out or discontinuation, or election not to take the AP Math exam? Given the findings in this study, researchers must pay close attention to the patterns among students from diverse racial groups in AP Math during Covid and examine the implications of gender. Taking the step after Covid to enroll in an in-person AP mathematics may be a greater challenge and choice than taking an AP Math Calculus BC after successful completion of AP Calculus AB. AP Math Calculus BC students are a different group in that they are more likely to include a wider range of STEM focused students whereas AB often includes Social Sciences students who want to satisfy a college math requirement before they enter college.

The findings from the present study looked at trends across year but is critical to establish baseline 2020 data on top achievers who are Black or from other underrepresented groups by examining AP Math Enrollment by race and gender and grade level. Of importance is the degree to which progress is being made or sustained to recruit and retain top achievers in AP Math. Additional research on the role of school and district level administrators in the provision of support systems to increase the success of talented students from underrepresented groups in AP programming would be beneficial to enable a much clearer picture of comprehensive support systems available to them. Change in policy and practice begins at the top, thus, school superintendents and board level policy makers have potential for additive impact on student achievement across the board as well as in advanced learner programs like AP (Bahar & Maker, 2011). Research on the role of district leaders in schools serving predominantly underrepresented student populations is also recommended.

Implications for Practice

Our findings indicate that despite the increasing access of AP coursework for talented students from historically unrepresented populations, their AP success rate remains low. Providing only simple pathways of access to AP programs such as payment for tests or access to classes ignores the complexity of the racial disparity within AP math participation. Although we do not have the data to support our claims, we underscore that recruitment and retention strategies precede student participation in AP math programs and help to create more sustained enrichment and high academic coursework during their elementary and middle school years, eliminating the lack of preparation necessary for secondary AP success. As prior research suggested (Chatterji, 2021; Finkelstein et al., 2012; Lubienski et al., 2004), increasing opportunities for these talented students to have early and sustained access to enrichment programs has potential to improve their success in AP coursework and on AP tests in secondary school. District level policy changes are recommended which create such experiences in schools that are predominantly Black, Hispanic, or Native American. Accessible programs coupled with improved professional development to address systemic bias and culturally responsive pedagogies for teachers working with talented students from unrepresented populations also have potential to improve student success rates. In addition to addressing access issues, it is important to use a critical lens to consider how schools and advanced programs will address the retention of these students in programs over time. Using strategies like early academic planning, support for exam readiness, mentoring, and an integration of a rich multicultural curriculum can also provide support for keeping students in programs and preparing them for rigorous coursework later on ((Bahar, 2013; Bahar & Maker, 2015; Ecker-Lyster & Niileksela, 2017). Ultimately, a program should evaluate the degree to which their programming options and policies support success or create barriers to students in their programs. Programs should make sure that any institutional barriers that can dissuade students from continued participation, as well as those barriers that thwart the efforts to recruit and retain high potential students are eliminated (Maker et al., 2021; Maker et al., 2022).

The findings from this study raise many questions about disparities, particularly for Black student recruitment and retention in higher level AP math. Educators should not be satisfied to only enroll Black students in the first of the AP math courses. Although there are many reasons that educators with high Black populations of gifted and advanced math learners struggle with recruitment and retention in advanced math courses, teacher referrals may possibly work as the greatest gatekeeper to AP Math (Chatterji, 2021). At this point we also leave several questions for future educators and researchers to be further investigated: Are Black students enrolled in AP Calculus AB being introduced to the value of continuing their math trajectory and encouraged to maximize their mathematical skills and opportunities? Are school leaders responsible for offering support for pre-AP programs and other preparatory models engaged in developing such models at schools where the AP student success rates are low? Are secondary educators, including district level administrators tapping into organizations and equity focused programming such as NABE to close racial gaps in math?

Although it is beyond the scope of our data and findings, we urge educators to be proactive responding to these continued excellence gaps. School district administrators and educators involved in course selection processes must begin to understand the gravity of the AP Math underrepresentation problem as it is reflective of the gifted and talented education problem that encouraged scholars, practitioners, policy makers, and other NAGC leaders have identified. There needs to be research examining the extent which AP Math teachers, counselors and school administrators are promoting culturally responsive anti-racist gifted and advanced programming to reduce the racial disparities. The processes undergirding access to AP course enrollment results in students from racially underrepresented populations being overlooked for AP course enrollment (Chatterji, 2021).

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Statement of Contribution of Researchers to the Article: KB wrote the introduction, methods, findings sections and created the figures and tables. KB, TG, MEH, and JLD interpreted the data and wrote the discussion sections. All authors revised the manuscript. All authors contributed to writing of this paper, read, and approved the final manuscript.

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