Who are the Mathematically Gifted? A Systematic Review of the Research on Cognitive Characteristics

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Abstract

This systematic review aimed to explore the cognitive characteristics of mathematically gifted individuals. Screening 497 studies, we obtained 22 empirical research that particularly explored cognitive characteristics of mathematically gifted individuals. We presented our findings under two major themes: domain-specific and domain-general abilities. Research that investigated domain-specific abilities suggested that problem-solving, mathematical creativity, and mathematical reasoning were essential characteristics of mathematical giftedness whereas some domain-general abilities including, perceptual abilities, visual-spatial ability, memory, and reasoning were found to contribute to mathematical giftedness. We also noted several within-group variations, suggesting a complex interaction of cognitive traits. Our results call for brand new frameworks on mathematical giftedness as the findings provide unique domain-specific characteristics that are different from what were provided by renowned models in the field. We also provided implications that encourage differentiated learning practices to meet the academic needs of mathematically gifted individuals.

Keywords: Mathematical Giftedness, Cognitive Characteristics, Visual-Spatial Ability, Reasoning, Systematic Review, Mathematical Talent.

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INTRODUCTION

For decades, researchers have widely examined the characteristics of gifted students. Although there has still been no consensus among researchers about the cognitive and behavioral outcomes of being gifted, they agreed that the concept of giftedness has been a multidimensional and dynamic construct (Johny, 2008; Renzulli, 1998; Singer et al., 2017), therefore the characteristics of gifted learners might look different and even vary across cultures, domains, and subjects (Galiullina & Mefodeva, 2020). Investigating how giftedness manifests itself in educational settings has been an important endeavor for researchers not only for research purposes, but also to provide educators with research-based information about the appropriate educational strategies and resources to meet the academic and social needs of gifted learners (Heller & Schofield, 2000).

Perspectives on Giftedness

Early giftedness research (e.g., Galton 1869; Spearman 1904; Terman 1954) viewed the G factor (G) as universal, overlooking domain-specific talents. Evolving research (e.g., Krutetskii 1976; Renzulli 1978; Stanley 1974) proposed intelligence as distinct abilities across domains, with Renzulli's (1978) three-ring model highlighting "above-average ability, high levels of task commitment, and high levels of creativity" (p. 8). Renzulli emphasized the interaction of these traits, rather than just superior intellectual abilities, to identify gifted behaviors accurately.

Together with this evolving research toward domain-specificity of giftedness, educators and policy makers began to recognize that giftedness could be multifaceted, with individuals exhibiting exceptional skills in areas such as mathematics, music, or language, independent of IQ (VanTassel-Baska, 2001). For instance, in Marland's report (1972), gifted students were identified as having potential in six areas: "specific academic aptitude, general intellectual ability, leadership ability, creative or productive thinking, visual and performing arts, and psychomotor ability" (p. 21). There is a growing emphasis on using multiple measures to identify giftedness rather than relying on a single criterion.

Our research, acknowledging the shift in gifted education towards domain-specific abilities, aims to review the recent studies to identify key cognitive characteristics of mathematically gifted students. Given the importance of understanding their unique needs to inform educational practices and programs (Leikin, 2021), the findings of this study will contribute to the literature by providing insights into the specific cognitive needs of mathematically gifted students, thereby enriching both the research and the practical applications in the education of mathematically gifted learners.

Mathematical Giftedness

Students with exceptional mathematical talent have been referred to by various terms in the literature, such as mathematically gifted (e.g., Leikin, 2010; Sowell et al., 1990; Sriraman, 2003), mathematically talented (e.g., Stanley et al., 1974), mathematically precocious (e.g., Lubinski & Benbow, 2006; Stanley, 1996), students with exceptional mathematical promise (e.g., Sheffield, 2006), and students with high mathematical ability (Krutetskii, 1976). Although these terms are often used interchangeably to describe the same group of students, there are subtle distinctions in what each phrase may represent.

Initially, mathematical giftedness was seen as an innate ability. This view changed significantly with the works of Vadim Andreevich Krutetskii and Julian Stanley. Krutetskii (1976) conducted pioneering research in Soviet society on the mathematical abilities of schoolchildren. He found that mathematically gifted children perceived the world through their "mathematical eyes" (p. 302), thinking mathematically and approaching problems with greater curiosity (Leikin, 2021). Krutetskii identified key elements in the development of mathematical giftedness: formalized perception of mathematical material, logical thought in quantitative and spatial relationships, rapid and broad generalization of mathematical objects, the ability to curtail reasoning processes, flexibility in mental processes (reversibility), and mathematical memory. His work provided a comprehensive understanding of the organization and development of mathematical abilities in schoolchildren.

These elements work together in harmony by affecting one another to make up a mathematical cast of mind, which helps mathematically gifted students perceive the world through a mathematical lens. Besides, the research mentioned some elements whose inclusion is optional in the structure of mathematical giftedness. The mathematical cast of mind is, nonetheless, affected by their growth and their existence or absence in the framework. Krutetskii (1976) presented them as "swiftness of mental processes," "computational abilities," "a memory for symbols, numbers, and formulas," "ability for spatial concepts," and "an ability to visualize abstract mathematical relationships, and dependencies." (p. 351). Most researchers have conducted their studies based on Krutetskii's conceptualization of mathematical giftedness (Leikin, 2021; Ozdemir & Bostan, 2021).

In the 1970s, Julian Stanley initiated the 50-year longitudinal Study of Mathematically Precocious Youth (SMPY) to understand the needs of mathematically gifted students who later excelled in STEM fields (Lubinski & Benbow, 2006). He also founded the Center for the Advancement of Academically Talented Youth (CTY) in 1979, independent of SMPY, to conduct talent studies and special education programs (Stanley, 1977; Stanley, 1982). The center conducted a three-year study to create a special "700-800 on SAT-M Before Age 13 Group," identifying 269 boys and 23 girls who scored at least 700 on SAT-M (Stanley, 1988). Observations indicated that their mathematical skills

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surpassed their verbal abilities, suggesting a significant intellectual capacity for quickly and effectively studying mathematics and related disciplines (Stanley, 1996).

The education and unique traits of mathematically gifted children, as well as approaches to enhancing their abilities, have been discussed (Leikin, 2021). These students are known for generating novel ideas, forming original associations, and effectively tackling unfamiliar challenges (Chang, 1985). They solve problems quickly, recall details, discern similarities and differences, and shift between abstract and concrete thinking rapidly (Wolfle, 1986). Their natural inclination towards problem-solving and systematic, logical thinking distinguishes them from non-gifted peers (Heinze, 2005; Sipahi, 2021). They display advanced critical mathematical thinking, superior memory skills (Leikin et al., 2013), and creativity (Leikin, 2021), with studies by Sriraman (2005) and Leikin et al. (2017a, 2017b) affirming a strong link between mathematical giftedness and creativity among professional mathematicians. This relationship suggests that superior mathematical skills often coincide with high creativity, encouraging further research into domain-specific cognitive traits in giftedness (Kontoyianni et al., 2013; Pitta-Pantazi et al., 2011).

Although previous research on giftedness has made significant contributions to our understanding of cognitive abilities, a clear gap remains concerning the domain specific cognitive characteristics that distinguish mathematically gifted individuals from their peers. This study addresses this gap by providing a comprehensive analysis of these domain-specific traits, which is expected to contribute to both theoretical frameworks and practical approaches in the education of mathematically gifted students.

Purpose and Significance of the Study

Teachers often struggle to address the needs of mathematically gifted students due to challenges in identifying and educating them (Archambault et al., 1993; Çapan, 2010; Sipahi, 2021; Ozdemir & Bostan, 2021). These students differ from their peers, necessitating teachers to deliver differentiated teaching strategies for those students (Koç Koca & Gürbüz, 2021; Krutetskii, 1976; VanTassel-Baska, 2001). However, there is a lack of comprehensive understanding of their cognitive characteristics. A recent analysis by Özdemir et al. (2024) highlighted trends like learning difficulties and neurocognitive traits in mathematically gifted students. To address this, we conducted a systematic review of studies from the past 10 years to inform more effective educational strategies.

METHODOLOGY

This study used a systematic review approach, chosen specifically to provide a thorough synthesis of existing literature on the cognitive traits of mathematically gifted individuals. By

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systematically evaluating empirical studies, this method allows for an organized analysis of both domain-specific and domain-general abilities within the targeted population.

The literature selection was conducted through two main channels: Web of Science (WoS) database and a library search engine of an R1 university in the U.S. which has access to most academic databases. WoS was chosen due to its rigorous indexing standards, ensuring that only high-quality, peer-reviewed research would be included in the analysis. The university library database was also selected to provide access to a wide array of academic journals and resources not always available in other databases, thus offering a comprehensive foundation for this review.

Google Scholar, although widely used by researchers, was excluded due to its limitations in filtering for quality. Unlike WoS, Google Scholar aggregates a vast range of sources, including non-peer-reviewed and non-academic documents, which may not meet the stringent criteria required for this systematic review.

Literature Selection Methodology

We employed a systematic review methodology, adhering to the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) guidelines to ensure research integrity and transparency. The focus was on peer-reviewed empirical studies from 2014 to 2024 that examined cognitive characteristics of mathematically gifted students.SA

Identification of Studies

Key phrases included "Math* AND (Gift* OR Talent* OR Precou* OR Exceptiona* OR Promis*) AND (character* OR trait* OR Skill* OR Styl*)," using the asterisk (*) as a wildcard for suffix variations. Logical operators like "AND" ensured focused retrieval of literature on the cognitive characteristics, traits, or skills of mathematically gifted individuals. The search was limited to peer-reviewed English-language articles to maintain scholarly quality and accessibility. The temporal scope covered publications from 2014 to 2024 to capture current research and developments over the past decade. On January 15, 2024, two researchers independently executed a search to mitigate biases, yielding 2,418 records.

To refine the initial results and identify the most pertinent studies, we used a systematic filtering method which added predetermined research areas (see Figure 1) as suggested by WoS. Through this process, 2,418 records were narrowed down to a targeted subset of 497 studies.

Article Eligibility Screening

Two researchers independently screened 497 records according to these inclusion criteria: (1) empirical studies focused on gathered data about the cognitive traits of mathematically gifted students; (2) studies with participants verified as mathematically gifted; (3) studies specifically examining the cognitive characteristics of these individuals, ensuring all selected literature directly contributed to our understanding of these attributes.

The selection procedure unfolded in two stages. Initially, the first and second authors independently assessed the eligibility of all 497 studies based on the inclusion criteria, resulting in 37 articles being preliminarily chosen by at least one author. This stage reached a 94% consensus on the eligibility of the studies. To resolve any disagreements, the authors held a meeting to discuss each discrepancy and align their understanding of the criteria. Following this, they independently re-evaluated the studies they initially disagreed on, eventually achieving unanimous agreement. This step confirmed there were no outstanding disagreements, leading to the inclusion of 19 studies in the systematic review. The complete selection process is illustrated in a PRISMA diagram (Figure 1).

Identification of Studies via Other Sources

Using the same rigorous methodology as in the initial screening, the search in the public R1 university database initially identified 465 studies. After removing duplicates, 384 studies remained for screening. Only 11 studies met our stringent inclusion criteria, selected by both researchers. Each study was examined for relevance, empirical quality, and focus on the cognitive characteristics of mathematically gifted students. These 11 studies were then compared with the 19 identified through the WoS search. After checking for duplicates, three additional unique studies were identified, resulting in a final total of 22 unique studies (see Figure 1). This process ensured the inclusion of journals not indexed in WoS.

Coding and Analysis

A thematic analysis was conducted with 22 empirical studies to identify recurring motifs related to the characteristics of mathematically gifted learners. Descriptive labels were assigned within each theme, and study outcomes were systematically categorized and cross-examined to discern patterns and cohesive themes. Initially, studies were coded under seven categories: problem-solving, creativity, spatial ability, verbal intelligence, working memory, speed of information processing (SIP), and reasoning. The second researcher suggested combining memory and processing speed with perceptual and reasoning skills and grouping verbal and non-verbal abilities into a single central theme. After refining these themes, the comprehensive coding of 22 studies resulted in two main themes: domain-general and domain-specific abilities.

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The coding process involved two independent researchers who initially categorized the studies into preliminary themes. To ensure reliability, an inter-rater agreement was calculated, with an initial agreement rate of 72.7%. Discrepancies were resolved through discussion, and a final coding framework was established with 100% agreement.

The themes identified were derived based on both prior literature and content analysis. Initial themes were guided by frequently discussed constructs in the literature, while subsequent analysis led to adjustments and refinement of these categories based on the content of the selected studies. This combined approach ensured that the themes reflected both established theoretical frameworks and the findings of this study.

FINDINGS

Cognitive characteristics were classified into domain-specific and domain-general based on their frequency and emphasis in the literature. Except for nine studies (Al-Hroub, 2020; Assmus & Fritzlar, 2022; Gonzalez et al., 2016; Paz-Baruch et al., 2016; Simut & Godor, 2022; Tjoe, 2015; Waisman et al., 2014; Waisman et al., 2016; Zhang et al., 2014), all others presented varied results from the same study across multiple cognitive ability sub themes.

Domain Specific Abilities (n = 17)

Three sub themes were identified within the domain-specific abilities: problem-solving and problem-posing, mathematical reasoning, and mathematical creativity.

Problem-Solving and Problem-Posing (n = 9)

Problem-solving and problem-posing skills were the most studied abilities (Gonzalez et al., 2016; Leikin et al., 2014, 2017a; Simut & Godor, 2022; Tjoe, 2015; Trinter et al., 2015; Waisman et al., 2014, 2016; Yazgan-Sag, 2022). Leikin et al. (2017a) found that mathematically gifted high schoolers preferred multiple proofs, explanations, and elaborated more frequently than their motivated peers. Similarly, academically gifted learners used more elaboration and control, but less memorization strategies compared to average-performing students (Simut & Godor, 2022). Additionally, gifted students used visual-spatial representations more often, positively correlating with their problem-solving proficiency (Leikin et al., 2014).

Similar results were observed in Trinter et al.'s (2015) study in which mathematically gifted elementary students demonstrated exceptional organizational skills during multi-step problem-solving, through their deep exploration of problems, and persistence in challenging contexts. However, some mathematically gifted high schoolers showed little attempt to apply Polya's (1945) fourth step in the solving process, which involves finding different ways to solve (Tjoe, 2015).

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Waisman et al. (2014, 2016) showed that investigated behavioral and electrophysiological characteristics when solving function-related problems, high school students who were identified as superiorly gifted in mathematics (S-MG) were significantly more accurate and quicker than other two groups of students; who were identified as generally gifted and excelling in mathematics (G-EM) and who excelled in mathematics but not identified as gifted (NG-EM). On the other hand, the accuracy level of NG-EM students was comparable to that of G-EM students in Waisman et al.'s (2014) study while this gap has widened even more in G-EM's favor in Waisman et al.'s (2016) study despite a longer reaction time. Waisman et al. (2016) added that a combination of EM and G (in S-MG and G-EM groups) seems to cause an increase in accuracy time which is an accumulative characteristic of S-MG students. Concerning these results, according to the prospective teachers, quick thinking was a key trait among mathematically gifted students. Nevertheless, a few teachers noted that lacking speed does not always equate to lacking mathematical giftedness (Yazgan-Sağ, 2022).

Despite its frequent association with problem-solving, problem-posing is regarded as a separate discipline. Gonzalez et al. (2016) examined the mathematical problem-posing abilities of 14-17-yearold mathematically gifted learners compared to their non-gifted peers. Gifted students posed more complex and varied problems, featuring a larger number of propositions, diverse numerical types, and requiring more complex steps, and unique calculation techniques. Their problems also demonstrated a greater range of semantic associations. These findings align with Yazgan-Sag (2022), who reported prospective teachers' views on mathematically gifted students.

Mathematical Reasoning (n = 4)

Some researchers observed high mathematical reasoning in mathematically gifted students (Haataja et al., 2020; Uclés et al., 2018; Yildiz & Durmaz, 2021; Zhang et al., 2020). For example, mathematically gifted high schoolers adeptly switched between different reasoning methods in approaching linear and nonlinear pattern generalizations differently (Yildiz & Durmaz, 2021). They used figural reasoning for linear patterns and numerical reasoning for nonlinear patterns, showcasing advanced generalization abilities (Yilmaz & Durmaz, 2021).

Adding a neuroscientific perspective, Zhang et al. (2020) found that mathematically gifted adolescents exhibited distinct neural advantages, such as enhanced temporal maintenance of topologies in the central executive network and right frontotemporal network. These networks are recruited more frequently and for longer durations compared to their peers, indicating that superior executive functions and a robust central executive control system are crucial for advanced mathematical thinking and reasoning abilities.

From an educational perspective, Uclés et al. (2018) suggested the use of assessment tasks that require complex reasoning skills during the identification and education of mathematically gifted.

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Because mathematically gifted learners were particularly distinguished from their non-identified peers in reasoning skills. Moreover, the tasks that required reasoning skills intensively were recognized as a proper platform to engage mathematically gifted learners during the learning process (Uclés et al., 2018). Furthermore, Finnish educators have identified two distinct types of giftedness: multifaceted talents, who impact society through social engagement, and singularly mathematically gifted students, who focus intensively on refining their mathematical reasoning abilities (Haataja et al., 2020). These findings suggest that mathematical reasoning is a key attribute of mathematical giftedness.

Mathematical Creativity (n = 8)

Assmus and Fritzlar (2022), Leikin et al. (2017a, 2017b), Trinter et al. (2015), Uclés et al. (2018), Yazgan-Sağ (2022), Yildiz and Durmaz (2021), and Zhang et al. (2020) provided complementary findings on the creativity of mathematically gifted students, highlighting their unique approaches and skills. Zhang et al. (2020) found that mathematically gifted university students were more likely to transition into networks associated with focused and adaptive information processing, indicating enhanced executive functions and creativity. Mathematical creativity, particularly originality, was a key feature. For instance, 10th and 11th-grade S-MG students excelled in flexibility and originality in solving arithmetic problems (Leikin et al., 2017b). Mathematically talented students often showed deep understanding and original thinking in their solutions (Uclés et al., 2018). Trinter et al. (2015) noted originality as a prominent trait in 2nd-grade mathematically promising students. Additionally, Leikin et al. (2017a) observed originality in the questions asked by gifted students during lessons.

Assmus and Fritzlar (2022) and Yıldız and Durmaz (2021) identified a strong correlation between mathematical giftedness and creativity, noting that mathematically gifted third graders exhibited greater creativity in handling figural patterns compared to non-gifted peers. Almost all gifted students demonstrated moderate flexibility or created original figural patterns, showing higher fluency than their non-gifted counterparts (Assmus & Fritzlar, 2022). Yıldız and Durmaz supported these results with a case where a high school student applied unconventional methods using Gauss's approach to generalize nonlinear patterns. Yazgan-Sağ (2022) further also supported this link, observing that prospective teachers recognized the importance of unique solution-finding as a key aspect of mathematical giftedness and creativity.

Domain General (n =11)

The findings were categorized within domain general abilities under six sub themes: perceptual skills, memory, processing speed, verbal, non-verbal, and reasoning abilities.

Perceptual Skills (n = 7)

Perception involves integrating, organizing, and responding to sensory information, which structures data from sensory receptors (Santos et al., 2020). It requires a complex interplay of motor, auditory, and visual skills. Motor skills support physical stability and movement confidence, foundational for writing and arithmetic. Auditory skills are crucial for listening, understanding instructions, and retaining information, essential for reading and spelling success. Visual skills help children understand and replicate visuals, affecting tasks like writing and arithmetic (Santos et al., 2020). Collectively, these skills are vital for attention and academic success, highlighting the deep connection between perceptual abilities and academic performance (Krieg, 1973).

Studies showed mathematically gifted students excel in visual, spatial skills, and attention control (Al-Hroub, 2021; Leikin et al., 2014, 2017b; Paz-Baruch et al., 2014, 2016; Ruthsatz et al., 2014; Uclés et al., 2018). Paz-Baruch et al. (2014) found that the G significantly influences visual-matching performance, highlighting the importance of visual abilities in mathematically gifted students. High school G-EM students showed superior visual-serial processing ability (Paz-Baruch et al., 2016). Leikin et al. (2014, 2017b) also emphasized the role of visual-spatial perception and matching abilities in mathematical excellence. Ruthsatz et al. (2014) noted that while math prodigies excelled in visual-spatial skills, art prodigies scored lower. Uclés et al. (2018) found that mathematically talented students aged 14-17 effectively used visualization, especially when combined with data organization and generalization skills, though no differences were observed in basic visual tasks like digit identification. Complex visual tasks required additional reasoning elements. Paz-Baruch et al. (2016) and Leikin et al. (2014, 2017b) also noted that attention control, including the ability to sustain attention and avoid errors, was a particular strength of S-MG students.

The findings were not very different for the twice exceptional students. In Al-Hroub's (2021) study, 80% of twice exceptional (2E) 5th and 6th grade learners had deficiencies in auditory skills or a combination of visual and auditory problems. However, mathematically gifted students with learning disabilities (MG/LDs) demonstrated high visual abilities. Specifically, their visual analysis skills appeared to be stronger than their visual engine integration and auditory skills.

Memory (n = 5)

Haataja et al. (2020) emphasized the concept of memory in a general sense that the Finland educators defined mathematical giftedness with the capacity to visualize, understand, and remember mathematical concepts quickly and clearly. Others (Kuo et al., 2014; Leikin et al., 2014, 2017b; Ruthsatz et al., 2014) have delved into the topic of memory, with specific emphasis on working, short term, and visual-spatial memory.

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Kuo et al. (2014) examined the relationship between high levels of mathematical and scientific talent combined with high IQ (MSTHIQ) and working memory capabilities. Their findings suggested that MSTHIQ students demonstrated superior working memory performance. This group outperformed not only their peers with average IQs within the same talent domains (MSTAIQ) but also typically developing students (TD) and students with Asperger's syndrome (AS), among individuals aged 16-26. This indicates a significant impact of combined intellectual abilities on working memory. Similar results in a wide age range were found by Ruthsatz et al.'s (2014) study. Among those participants (age 6 to 32), the music and math prodigies displayed superior working memory skills, while the art prodigies did not score as high. In the study conducted by Leikin et al. (2014), while high school S-MG students were better than G-EM students and NG-EM students in the working memory, G-EM and NG-EM groups' performances on the working memory were similar. On the other hand, G-EM students displayed superior short-term memory skills compared to their NG-EM counterparts. A later study (Leikin et al., 2017b) found no significant difference in visual-spatial memory capability between mathematically gifted and non-gifted high school students.

Processing Speed (n = 3)

Speed of information processing (SIP) has been found to be closely associated with the combination of G and EM factors, as suggested by Paz-Baruch et al. (2014). Kuo et al. (2014) also underscored the advantage in processing speed associated with mathematical and scientific talent, particularly when coupled with high IQ because the MSTHIQ group demonstrated superior processing speed compared to the other groups, followed by the MSTAIQ group, then the TD group, with the AS group showing the lowest processing speed. In the same vein, Leikin et al. (2014) found significant differences among the groups in the SIP in some specific areas. Both the S-MG and G-EM students exhibited higher accuracy and speed scores on these tasks compared to NG-EM students.

Verbal abilities (n = 3)

Verbal ability was one of the cognitive abilities found related to mathematical giftedness in Kuo et al.'s (2014) and Al-Hroub's 2021) studies, with a particular emphasis on skills such as information, similarities, arithmetic, vocabulary, and comprehension. Kuo et al. suggested that MSTHIQ students aged 16–26 possessed exceptional verbal skills. Moreover, the arithmetic area was the highest specifically for the MSTHIQ group. The findings were congruent with the twice exceptional groups. These same areas were noticeably superior in both categories of students in 5th and 6th grade with learning disabilities, including those MG/LDs and those who were not. The MG/LDs sample demonstrated a particular proficiency in vocabulary (Al-Hroub, 2020). A year later, they reached comparable results that 2E learners demonstrated high verbal abilities (Al-Hroub, 2021).

Non-Verbal Abilities (n = 2)

Kuo et al. (2014) found that mathematically and scientifically talented students with average IQs had weaker non-verbal abilities compared to their high-IQ peers. Al-Hroub (2020) further analyzed non-verbal skills in twice-exceptional individuals through tasks like picture completion, arrangement, coding, block design, and object assembly. Students with both MG/LDs and average-IQ/LDs were more proficient in verbal than non-verbal skills, with coding and picture arrangement scoring the lowest in both groups.

Reasoning Abilities (n = 2)

Researchers (Ruthsatz et al., 2014; Zhang et al., 2014) explored the multifaceted nature of reasoning, focusing on its quantitative, fluid, and deductive aspects. Math and music prodigies excelled in quantitative reasoning, unlike art prodigies. While math prodigies outperformed art prodigies in fluid reasoning, music prodigies showed no significant differences in reasoning abilities compared to the others (Ruthsatz et al., 2014).

In Zhang et al.'s (2014) study, deductive reasoning was among general cognitive abilities associated with mathematically gifted learners. Moreover, the group of mathematically gifted individuals aged 15-18 outperformed the control group in average response accuracy in fluid reasoning, indicating a significant group difference where the mathematically gifted adolescents responded more accurately.

Discussion

Among the various cognitive characteristics, problem-solving stood out as a key characteristic of mathematically gifted students, who use diverse strategies and representations (Leikin et al., 2014; Simut & Godor, 2022) which was mainly observed in the gifted literature (Akdeniz & Alpan, 2020; Bayazıt & Koçyiğit, 2017; Budak, 2012; Pativisan, 2006; Sowell et al., 1990; Yıldız et al. 2012). Besides, their complex problem-solving abilities included inclination towards in-depth exploration (Leikin et al., 2017a), enduring interest in demanding contexts, and a desire to understand mathematical concepts (Trinter et al., 2015). This finding implies that outstanding problem-solving abilities were not solely dependent on implementing different strategies, but also a significant difference in how the mathematically gifted approached problems (Gorodetsky & Klavir, 2003; Montague, 1991).

Several studies reviewed linked mathematical giftedness with high creative performance, particularly highlighting originality (Leikin et al., 2017a, 2017b; Uclés et al., 2018) and flexibility (Leikin et al., 2017b) as key elements of mathematical creativity. Our results confirmed this prevailing view. Renzulli's (1978) three-ring model of giftedness, which includes creativity, supports this fact.

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From a domain-specific perspective, Krutetskii (1976) also cited flexibility and originality as notable features of mathematical giftedness. The close connection between mathematical giftedness and creativity, especially in problem-solving, is recognized as a fundamental indicator of mathematical giftedness (Assmus & Fritzlar, 2022; Trinter et al., 2015; Uclés et al., 2018; Yildiz & Durmaz, 2021). These findings suggest that problem-solving and creativity are inextricably linked, as engaging in problem-solving allows individuals to showcase their creativity through diverse strategies. Given the problem-centric nature of mathematics, solving problems is crucial for demonstrating creativity. Therefore, educators should craft and implement problem-solving activities and curricula that promote originality and flexibility to foster and nurture creativity.

Our findings largely align with prominent frameworks on giftedness, such as Renzulli's (1978) Three-ring Conception of giftedness which posits that giftedness emerges from a combination of aboveaverage ability, high task commitment, and creativity, while also providing a basis for new paradigms that uniquely differ from established models. Our study confirmed this approach by highlighting the interactions of cognitive characteristics such as creativity, problem-solving, and other domain-specific and general abilities in mathematically gifted students. However, perceptual skills, which played a significant role in mathematical giftedness, were not identified as clusters of giftedness in Renzulli's model. Renzulli may have included these within Above Average Abilities, which covers general and specific abilities, including spatial relations and memory. While proposing perceptual abilities as a fourth ring is beyond our data's scope, future research was suggested to consider a domain-specific model of mathematical giftedness that includes perception and information processing. This aligns with other research linking high spatial ability and visual processing with high achievement in STEM (Bahar & Maker, 2020; Bahar et al., 2021; Benbow et al., 2000; Maker, 2020; Maker et al., 2021; Shea et al., 2001; Wai et al., 2009).

Our findings also align with Krutetskii's (1976) domain-specific framework on mathematical giftedness, like Renzulli's model, emphasizing the importance of a multifaceted approach to high mathematical abilities. Our findings have demonstrated that the possession of mathematical giftedness is not constrained to a mere cognitive characteristic such as intelligence, but rather encompasses a multi-dimensional array of diverse and complex. Having said that, Krutetskii also posited that the presence or absence of some cognitive characteristics such as speed, memory, visual and spatial abilities should not be considered compulsory markers of such giftedness, which was different from our findings. In contrast, a significant portion of our research supports the notion that some domain general abilities (e.g., visual, spatial ability, and memory) serve as indicators of mathematical giftedness. Although these results do not oppose Krutetskii's acknowledgment of potential variability within the constellation of skills associated with mathematical giftedness, our results also suggest further research to explore the significance of these cognitive abilities and their potential contribution to mathematical giftedness.

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Our review highlights increasing attention to students who are both mathematically gifted and have learning disabilities, particularly in studies from 2020. This indicates the urgency for educators to address the unique needs of twice-exceptional students. As this research area is still developing, primarily driven by Al-Hroub (2020, 2021), further exploration is needed. Future studies on twice-exceptionality are expected to enhance understanding of the cognitive profiles of mathematically gifted students.

Implications for Research

This study invites our readers to consider several observations from our reviewed studies, focusing on research designs and measurements used to identify mathematical giftedness. Both qualitative and quantitative methods were equally preferred, while mixed methods were used in only four studies. This indicates a possible under recognition of the mixed method's advantages or its application difficulties. Future researchers are encouraged to use mixed methods to provide deeper insights into the complex structure of mathematical giftedness by combining quantitative and qualitative findings.

Most reviewed studies favored using multiple selection criteria to assess mathematical giftedness, highlighting its complex structure (Table 1). Common tools included WISC, WAIS-III, Raven, and Stanford-Binet tests (Al-Hroub, 2020, 2021; Kuo et al., 2014; Leikin et al., 2014, 2017a, 2017b; Paz-Baruch et al., 2014, 2016; Ruthsatz et al., 2014; Waisman et al., 2014, 2016; Yildiz & Durmaz, 2021; Zhang et al., 2014, 2020). Following this, performance in math class, math grades, and overall mathematical proficiency were commonly preferred (Assmus & Fritzlar, 2022; Kuo et al., 2014; Leikin et al., 2014, 2017a, 2017b; Paz-Baruch et al., 2014, 2016; Simut & Godor, 2022; Waisman et al., 2014, 2016; Yildiz & Durmaz, 2021; Zhang et al., 2014). Among domain-specific tests, the Scholastic Assessment Test in Mathematics (SAT-M) was the most common tool (Leikin et al., 2014, 2017b; Paz-Baruch et al., 2014, 2016; Tjoe, 2015; Waisman et al., 2014, 2016), compared to others like the Dynamic Assessment of Mathematical Achievement (Al-Hroub, 2020, 2021; Gonzalez et al., 2016; Uclés et al., 2018). This may be due to limited access to domain-specific assessments. Despite the multidimensional nature of mathematical giftedness, domain-specific tests are powerful tools. Our findings show that mathematically gifted students commonly exhibit high proficiency in domain-specific reasoning abilities, such as numerical, figural (Yildiz & Durmaz, 2021), and mathematical reasoning (Haataja et al., 2020; Uclés et al., 2018; Zhang et al., 2020). Krutetskii (1976) and Stanley (1974) argued for domain-specific assessments in identifying mathematical giftedness. Our findings support evaluating mathematical reasoning as a core component in this process, a well-accepted sentiment across numerous studies (e.g., Benbow et al., 2020; Sowell et al., 1990; McCabe et al., 2020). Future research should examine the effectiveness of various assessment tools and explore integrated approaches to better understand mathematical giftedness.

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Teacher nomination has become a preferred criterion for determining mathematical giftedness, alongside high performance in mathematics (Al-Hroub, 2020, 2021; Assmus & Fritzlar, 2022; Leikin et al., 2014, 2017b; Tjoe, 2015; Trinter et al., 2015; Waisman et al., 2014, 2016; Yildiz & Durmaz, 2021; Zhang et al., 2014, 2020). Teachers can observe students' classroom behavior, participation, and problem-solving approaches, which are as important as math performance or grades. This may reflect students' academic achievement and other characteristics like creativity, problem-solving, and reasoning. However, the subjective nature of this approach and inconsistencies between teachers may introduce bias. Future research should focus on the reliability and objectivity of teacher recommendations and on enhancing teachers' awareness and training in recognizing gifted students.

Research has primarily focused on secondary school students, potentially neglecting early identification and support for younger gifted students (Table 2). This trend might exist because it's more convenient to study older students, whose cognitive characteristics and understanding of complex mathematical concepts are more explicit. Future studies should investigate younger students to identify early signs of giftedness and develop strategies to support their growth. Educational policies should prioritize identifying and supporting gifted students across all age groups.

Limitations

This study has some limitations regarding its choice of data sources as we only included studies from WoS and a major university library database. While WoS is known for its strong standards and high-quality indexing, it may not capture all relevant research, especially from newer or less established journals. Similarly, the exclusion of Google Scholar, which encompasses a broader range of studies, was intended to maintain quality but may have limited the scope of findings. Future studies could benefit from including additional databases, such as Google Scholar or Scopus, to provide a more comprehensive review.

Conclusion

This study offers a comprehensive cognitive profile of mathematically gifted individuals, covering both domain-specific and domain-general abilities. Research into mathematical giftedness began in the early 1900s and has since grown into a dynamic field with contributions from diverse researchers. Our findings support the notion that mathematical giftedness is a complex, multi-faceted phenomenon, suggesting further explorations into its nature. Highlighting the importance of addressing the academic needs of these students, our study advocates for future practices that involve the development and use of effective identification tools, educational materials, and teacher training programs, all grounded in contemporary research.

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Appendices

Table 1 Objective(s), Measure(s), and Study Design of the Reviewed Literature

Author(s) / Date	Objective(s)	Measure(s) of Mathematical Giftedness	Study Design
Al-Hroub (2020)	To investigate cognitive characteristics of twice exceptional students or mathematical gifted students with learning disabilities (MG/LD).	Teacher nomination, IQ score 120 or above on the Wechsler Intelligence Scale for Children – the third Jordanian version (WISC-III-Jordan), high mathematical abilities in different areas (calculation operations, ordering of decimals, rounding up, geometry, algebra, and problem-solving) of Dynamic assessment (DA) test and high variance of performance between the pre-and post-DA test.	Quantitative study
Al-Hroub (2021)	To examine the utility of psychometric and DA (Dynamic Assessment) tests for identifying cognitive and perceptual characteristics of twice exceptional students or mathematical gifted students with learning disabilities (MG/LD) in Jordan.	Teacher nomination, IQ score 120 or above on the Wechsler Intelligence Scale for Children – the third Jordanian version (WISC-III-Jordan), high mathematical abilities in different areas (calculation operations, ordering of decimals, rounding up, geometry, algebra, and problem-solving) of Dynamic assessment (DA) test and high variance of performance between the pre-and post-DA test.	Qualitative study with multiple cases
Assmus & Fritzlar (2022)	To evaluate if mathematically gifted primary school students vary from non-gifted ones in high creativity in working with mathematical patterns and structures.	Teacher nomination, math grades in the school, and university entrance test.	Qualitative study with semi- standardized individual interviews
Gonzalez et al. (2016)	To describe, analyze, and characterize the capability of mathematically talented students in performing tasks related to posing arithmetic word problems.	Participation in the Stimulation of Mathematical Talent Andalusian Project (ESTALMAT) and a specific math test.	Mixed method study
Haataja et al. (2020)	To investigate the perspectives of educators on how a social learning atmosphere can cater to the social-emotional needs of mathematically talented youth.	Teacher Perception	Qualitative study with semi- structured interviews and observations

Author(s) / Date	Objective(s)	Measure(s) of Mathematical Giftedness	Study Design
Kuo et al. (2014)	To investigate the cognitive profiles and social integration of mathematically and scientifically talented (MST) and those with Asperger's syndrome (AS) contrasted to typically developing (TD) students.	A score above the 97th percentile on the Wechsler Adult Intelligence Scale, 3rd edition (WAIS-III), and studied or have been studying in a class of mathematics and science in senior high school.	Quantitative study
Leikin et al. (2014)	To explore the cognitive abilities of students who excel in mathematics, with varying levels of giftedness (G-EM and NG- EM) and superior performance (S-MG).	Teacher nomination, a score above 27 (out of 30) on the Raven's Advanced Progressive Matrix Test (RAPMT), a score above 26 (out of 35) on the Scholastic Assessment Test in Mathematics (SAT-M), studying mathematics at high level mathematics class with a score above 90, demonstration of exceptional achievements in mathematics: Membership in national Olympiad teams (in mathematics or computer sciences) or studying university mathematics courses parallel with their high school studies, and attaining scores above 95.	Quantitative study
Leikin et al. (2017a)	To explore the types of questions that students of two different classes: Gifted Class (GC) and Motivation Class (MC) ask and to identify the differences between their questions.	IQ score 130 or above (tested in the 3rd grade) and studying mathematics at a high level for either their enjoyment of mathematics or by the idea that mathematics is important for their future careers.	Qualitative study
Leikin et al. (2017b)	To uncover particular traits of mathematically gifted students in three areas: primary cognitive abilities, neuro-cognitive characteristics, and mathematical creativity.	Teacher nomination, IQ score above 130, score above 90 (out of 100) in mathematics at a high level (the highest of the three levels studied in secondary school in Israel), a score 27 (out of 30) on the Raven's Advanced Progressive Matrix Test (RAPMT), a score above 26 (out of 35) on the Scholastic Assessment Test in Mathematics (SAT-M), demonstration of exceptional achievements in mathematics: Membership in national Olympiad teams (in mathematics or computer sciences) or studying university mathematics courses parallel with their high school studies, and attaining scores above 95.	Quantitative study

Author(s) / Date	Objective(s)	Measure(s) of Mathematical Giftedness	Study Design
Paz-Baruch et al. (2014)	To examine the connection between speed of information processing (SIP), general giftedness (G), excellence in mathematics (EM), and gender factors.	IQ score above 130, a score 27 (out of 30) on the Raven's Advanced Progressive Matrix Test (RAPMT), a score above 26 (out of 35) on the Scholastic Assessment Test in Mathematics (SAT-M), and studying mathematics at a high level with scores above 90 (out of 100).	Mixed method study
Paz-Baruch et al. (2016)	To examine the visual processing abilities associated with both general giftedness (G) and excellence in mathematics (EM) for gifted students.	IQ score above 130, a score 27 (out of 30) on the Raven's Advanced Progressive Matrix Test (RAPMT), a score above 26 (out of 35) on the Scholastic Assessment Test in Mathematics (SAT-M), studying mathematics at a high level with scores above 90 (out of 100).	Mixed method study
Ruthsatz et al. (2014)	To investigate whether child prodigies' cognitive profiles differ significantly across various domains (art, music, and math) and to explore the underpinnings of prodigious abilities.	Stanford-Binet 4th and 5th edition only with two parts: the fluid reasoning and working memory, national or international acclaim during adolescence, news reporting and by way of referral.	Quantitative study
Simut & Godor (2022)	To investigate student approaches to studying via questionnaires focusing on non-observable constructs.	Rank in the 95th percentile for all plausible math scores in PISA.	Quantitative study
Tjoe (2015)	To explore the aesthetic perceptions of expert mathematicians and mathematically gifted high school students regarding their preferred approaches to mathematical problem-solving.	Advanced Placement (AP) Calculus teachers' nomination, participation in AP Calculus course, and a score 717 and above (out of 800) on the Scholastic Assessment Test in Mathematics (SAT-M).	Qualitative study
Trinter et al. (2015)	To investigate if the tasks proposed by Sheffield to identify mathematical promise could be shown to manifest in mathematically promising elementary students within the Problem- Based Learning environment.	Teacher perception and teacher nomination (Talent Identification Protocol)	Qualitative study
Uclés et al. (2018)	To investigate how mathematically talented students recognize the pattern upon which the Braille code is built and how they create their own mathematical language for visually impaired people.	Participation in the Stimulation of Mathematical Talent Andalusian Project (ESTALMAT) and a specific math test.	Qualitative study

Author(s) / Date	Objective(s)	Measure(s) of Mathematical Giftedness	Study Design
Waisman et al. (2014)	To investigate how general giftedness and excellence in mathematics influence students' ability to translate between graphical and symbolic representations of functions.	Teacher nomination, IQ score above 130 (tested in the 3rd grade), a score above 28 (out of 30) on the Raven's Advanced Progressive Matrix Test (RAPMT), a score above 26 (out of 35) on the Scholastic Assessment Test in Mathematics (SAT-M), studying mathematics at a high level mathematics class with a score above 90, demonstration of exceptional achievements in mathematics: Membership in International Computer Science Olympiad team or studying university mathematics courses parallel with their high school studies, and attaining scores above 95.	Quantitative
Waisman et al. (2016)	To identify similarities and differences in behavioral and electrophysiological measures among these groups of students.	Teacher nomination, IQ score above 130 (tested in the 3rd grade), a score above 28 (out of 30) on the Raven's Advanced Progressive Matrix Test (RAPMT), a score above 26 (out of 35) on the Scholastic Assessment Test in Mathematics (SAT-M), studying mathematics at high level mathematics class with a score above 90, demonstration of exceptional achievements in mathematics: Membership in International Computer Science Olympiad team or studying university mathematics and computer courses in the university level parallel with their high school studies, and attaining scores above 95 (out of 100).	Quantitative
Yazgan-Sag (2022)	To reveal prospective primary mathematics teachers' views about mathematical giftedness and the characteristics of mathematically gifted students.	Teacher Perception	Qualitative study using focus group interviews.
Yildiz & Durmaz (2021)	To investigate how a mathematically gifted student applies generalization strategies to recognize linear and nonlinear patterns in a matchstick problem.	Teacher nomination, IQ score 130 and above on the Wechsler Intelligence Scale for Children–Revised Form (WISC-R), a high score in Turkish High School Entrance Exam, and high performance in mathematics and physics at school	Qualitative study specifically phenomenology with task-based interview

Author(s) / Date	Objective(s)	Measure(s) of Mathematical Giftedness	Study Design
Zhang et al. (2014)	To explore the functional binding in the crucial fronto-parietal network of reasoning in mathematically gifted adolescents.	Teacher nomination, a score above 32 on the Raven's Advanced Progressive Matrix Test (RAPMT) and high academic performance (having prizes in nationwide or provincial mathematical competitions).	Mixed method study
Zhang et al. (2020)	To investigate the unique brain activity patterns of mathematically gifted adolescents as they engage in complex reasoning tasks.	Teacher nomination, a score above 32 on the Raven's Advanced Progressive Matrix Test (RAPMT) and award- winning experience in at least one of some important mathematical competitions (e.g., Math Olympic Competition).	Quantitative

Authors / Date	Sample(s)	Finding(s)
Al-Hroub (2020)	N=52 (5th and 6th grade). 30 (16 female and 14 male) mathematically gifted students with learning disabilities (MG/LD) and 22 (10 female and 12 male) average IQ/LDs students in Amman, Jordan.	In perceptual skills, MG/LDs performed better than the Average-IQ/LDs group, while the Average-IQ/LDs group demonstrated a slight edge in spatial and visual ability.
Al-Hroub (2021)	N=52 (5th and 6th grade). 30 (16 female and 14 male) mathematically gifted students with learning disabilities (MG/LDs) and 22 average IQ/LDs students in Amman, Jordan.	The data presented that MG/LDs had exceptional visual and verbal capabilities. Analyzing learning ability areas separately revealed that MG/LDs learners had notably higher average visual short-term memory skills than auditory short-term memory skills (which were still below average).
Assmus & Fritzlar (2022)	N=24 (3rd grade). 14 (One female and 13 male) mathematically gifted and 10 (Two female and eight male) non–gifted students.	All the mathematically gifted students, save for one, were either moderately flexible or created at least one original figural pattern.
Gonzalez et al. (2016)	N=40. 21 (13 to 15 years old) mathematically talented and 19 (14 to 15 years old) standard group of students.	Talented students' problems involved a greater number of propositions, used different types of numbers, required more steps and different calculation processes to solve, and had a higher number of different semantic relationships.
Haataja et al. (2020)	N=5. Three teachers, a principal, and a school social worker.	Finland educators thought that mathematically gifted students channeled their efforts into one talent, exhibiting a great dedication to mastering mathematical reasoning skills. They also defined mathematical giftedness with the capacity to visualize, understand, and remember mathematical concepts quickly and clearly.

Authors / Date	Sample(s)	Finding(s)
Kuo et al. (2014)	N=84 (16 to 26 years old) (All male). 24 Mathematically and scientifically talented students with a high IQ (MSTHIQ), 17 mathematically and scientifically talented students with an average IQ (MSTAIQ), 14 students with Asperger's syndrome (AS), and 29 typically developing (TD) students.	MSTHIQ students exhibited the greater performance on processing speed, some verbal and non-verbal abilities followed by MSTAIQ students, TD students, and students with AS. Mathematical and scientific talent with a high IQ played a significant role in improving working memory capability.
Leikin et al. (2014)	N=56 (10th and 11th grade) (All male). Mathematically gifted (S-MG), generally gifted students excelling in school mathematics (G-EM), and students excelling in school mathematics without being identified as gifted (NG-EM).	S-MG students exhibited superior performance on working memory, and specifically in visual-matching, indicating a higher accuracy and speed compared to NG-EM students. Attention tasks, as evidenced by fewer errors made by S-MG students compared to NG-EM students.
Leikin et al. (2017a)	N=50. Twenty-two (10th grade) mathematically gifted and 28 (9th grade) highly motivated students.	Mathematically gifted students' elaboration questioning exemplified their yearning to make sense of the ideas presented by others, and discuss their own ideas during problem-solving. They displayed exceptional creativity and originality in their mathematical thinking by posing questions that were not directly tied to the material, yet still relevant to the topic at hand.
Leikin et al. (2017b)	N=50. (10th and 11th grade). Generally gifted (NG-EM), generally gifted excelling in mathematics (G-EM) students, and students with superior performance in mathematics (S-MG). <i>The number of participants varied from test to test.</i>	They highlighted S-MG's exceptional flexibility, originality, and creativity. All participants had an analogous capability on the visual-spatial memory task; however, S-MG and G-EM students exhibited significantly higher performance than NG-EM students on the Visual matching task, and S-MG students exhibited fewer mistakes than NG-EM students on the Attention task.

Authors / Date	Sample(s)	Finding(s)
Paz-Baruch et al. (2014)	N=190 (10th and 11th grade). Students identified as generally gifted and excelling in mathematics (G–EM), students identified as generally gifted but not excelling in mathematics (G–NEM), students excelling in mathematics but not identified as generally gifted (NG–EM), and students not identified as generally gifted or excelling in mathematics (NG–NEM).	The visual-matching performance of students is significantly impacted by the general intelligence (G) factor. Also, the G–EM group outperformed the other three groups on all five SIP tasks.
Paz-Baruch et al. (2016)	N=190 (10th and 11th grade). Students identified as generally gifted and excelling in mathematics (G–EM), students identified as generally gifted but not excelling in mathematics (G–NEM), students excelling in mathematics but not identified as generally gifted (NG–EM), and students not identified as generally gifted or excelling in mathematics (NG–NEM).	The research revealed that the general intelligence (G) factor had a significant impact on students' attention capacity. Visual-Serial processing is a strong trait for G-EM students.
Ruthsatz et al. (2014) Simut & Godor (2022)	N=18 (Six to 32 years old) (Five female and 13 male). Eight music, five math, and five art prodigies. N=4,929 (15 years old). 247 (121 female and 126 male) gifted and 4,682 (2299 female and 2383 male) average students.	Music and math prodigies outperformed art prodigies in working memory, with math prodigies showing the greatest visual-spatial abilities. Both math and music prodigies excelled in quantitative reasoning without significant differences between them. In fluid reasoning, math prodigies passed art prodigies, but no significant differences were observed between math and music prodigies or between music and art prodigies. The gifted learners relied heavily on elaborate and well-controlled approaches, while significantly reducing their reliance on memorization when compared to their average peers.
Tjoe (2015)	N=57. Three expert mathematicians and 54 (11th and 12th grade) (28 female and 26 male) mathematically gifted students.	Gifted students did not prefer to use Polya's (1945) fourth step, called looking back, in the problem-solving process which requires finding different solution ways than the usual ones.
Trinter et al. (2015)	N=8. Three teachers of second graders (all female) and five (2nd grade) (Four female and one male) mathematically promising students.	Exceptional organizational skills and a dedication to multi-step problem-solving were evident in the students' thorough exploration of problems, perseverance in challenging situations, and curiosity about the connections between mathematical concepts. Also, they demonstrated creativity in their approach to problem-solving.

Authors / Date	Sample(s)	Finding(s)
Uclés et al. (2018)	N=74. 37 (14 to 17 years old) (13 female and 24 male) mathematically talented students and 37 non-talented students.	The mathematically gifted students exhibited a thorough comprehension of the problem's internal structure and showcased original thinking in their solutions. They utilized a combination of visualization, data organization, and generalization skills. Mathematical reasoning was suggested as one of the identification tools for mathematical talents.
Waisman et al. (2014)	N=84 (Grade 10 and 12; 16 to 18 years old) (All male). 19 generally gifted and excelling in mathematics (G-EM), 21 generally gifted but do not excel in mathematics (G-NEM), 16 not generally gifted but are excelling in mathematics (NG-EM), 19 neither generally gifted nor excelling in mathematics (NG-NEM), and nine super mathematically gifted students (S- MG).	Excellence in mathematics significantly affects problem-solving performance, reflected in both accuracy and reaction time. G-EM and NG-EM showed similar levels of accuracy, but NG-EM students took longer to respond. S-MG were more accurate and quicker in solving tasks than G-EM and NG-EM groups.
Waisman et al. (2016)	N=74 (Grade 10 and 12; 16 to 18 years old) (All male). 16 generally gifted and excelling in school mathematics (G-EM), 19 generally gifted but did not excel in mathematics (G-NEM), 17 excelled in mathematics but were not identified as generally gifted (NG-EM), 15 neither generally gifted nor excelling in mathematics (NG-NEM), and seven super mathematically- gifted (S-MG).	G-EM demonstrated the highest accuracy in problem-solving among all groups. Specifically, when comparing the three groups that excelled in mathematics, S-MG students exhibited even higher accuracy than G-EM students. The study considers accuracy as an accumulative characteristic of S-MG students, as increases in accuracy from NG-EM to G-EM and from G-EM to S-MG students were non-significant.
Yazgan-Sag (2022)	N=11 Prospective primary mathematics teachers.	Many participants accepted that mathematically gifted students demonstrate quick- thinking, not always, and grasping abilities. Participants stated that mathematically gifted students tended to demonstrate reasoning, find unique solutions, show abstract thinking, and have abilities of complex problem-solving and problem-posing.
Yildiz & Durmaz (2021)	N=1 (10th grade) (male) mathematically gifted student.	In the linear pattern, they generally used figural reasoning for generalizing the patterns. In nonlinear patterns, they solely used numerical reasoning.

Authors / Date	Sample(s)	Finding(s)
Zhang et al. (2014)	N= 24. 11 (15 to 18 years old) (Three female and eight male) mathematically gifted and 13 (14 to 16 years old) (five female and eight male) non-gifted students.	In terms of fluid reasoning, the mathematically gifted students performed better than the control group with a higher average response accuracy. A noticeable distinction between the two groups was observed as the math-gifted students displayed shorter reaction times and higher accuracy in their responses compared to the controls.
Zhang et al. (2020)	N=26, 12 (15 to 17 years old) (Three female and nine male) mathematically	Deductive reasoning was recognized as a key cognitive ability linked to mathematical giftedness.
Zhàng et al. (2020)	gifted and 14 (14 to 16 years old) (Six female and eight male) non-gifted students.	mathematically gifted students. The significance of improved executive functions and central executive control for advanced mathematical thinking and reasoning was mainly underscored.

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Figure 1. Prisma Diagram of Search Process



Note. *The three categories of search terms: Math* AND (Gift* OR Talent* OR Precou* OR Exceptiona* OR Promis*) AND (character* OR trait* OR Skill* OR Styl*). **Research areas: Education, Educational Research, Psychology, Neurosciences, Neurology, Rehabilitation, Social Sciences, Other Topics, Mathematical Methods in Social Sciences, Development Studies, and Cultural Studies.