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Variations in Thinking Levels Among Senior High School Students Across Different Study Programmes

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Abstract

The study investigated whether senior high school students' thinking levels in permutation and combination differ by the programme of study. Therefore, the Structure of the Observed Learning Outcome (SOLO) taxonomy was used as a theoretical framework to assess their thinking levels in permutation and combination. Quantitative research method that employed descriptive research design was used as a strategy of enquiry in this present study. Three senior high schools were purposively selected and a sample of 360 students which comprised 256 males and 104 females were randomly selected for the study. The data were collected using tests. The data were analysed using descriptive statistics (percentages, mean and standard deviation) and inferential statistics (Kruskal-Wallis tests). The results indicated that, the majority (73.9%) of the students reached the lower levels of the SOLO taxonomy (pre-structural, uni- structural and multi- structural) while a few (26.1%) reached the higher levels (relational and extended abstract). Furtherance to this, the Kruskal-Wallis H test indicated that there was a statistically significant difference in the thinking levels of the SOLO taxonomy across the various programme of study where General Science students differed significantly from General Agriculture and Business students. Therefore, it is recommended that educators should use differentiated instructional methodologies, including active learning techniques adapted to each subject of study. Again, to overcome the identified disparities, curriculum developers may incorporate discipline-specific cognitive skill-building activities.

Keywords: Combinatorial, SOLO taxonomy, Permutation

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INTRODUCTION

In the face of 21st-century challenges, every individual should have the thinking skills to solve every problem in mathematics or at the highest level of thinking. When an individual think of solving a mathematical problem, it means that the person is doing mathematical thinking. Schoenfeld and Sloane (2016) state that mathematical thinking is the ability to think and make judgments independently. Mathematical thinking, according to Sumarmo (2010) involves thinking about mathematics (doing math) or solving complex and simple mathematical problems. The process of mathematical thinking involves translating incoming external information into symbols that are then translated into calculations in accordance with established mathematical rules. Moreover, mathematical thinking is an integral part of learning about and learning through mathematics. It entails more than just acquiring a set of skills. Students engage in mathematical thinking by identifying and posing issues, as well as selecting and implementing relevant techniques to solve them. Conjecturing and proving, applying and verifying, generalizing, employing mathematical models, conveying ideas and answers, and reflecting on learning are all part of the process (PISA, 2006).

Having high mathematical thinking helps students solve mathematics problems with low mathematics anxiety and show positive attitudes towards mathematics (Kargar, Tarmizi & Bayat, 2010). Therefore, in many nations, the ability to think mathematically and solve problems based on mathematical reasoning is a crucial educational goal. In line with this, Ghanaian educational system has made mathematics a compulsory subject studied in primary education, secondary education, and higher education because mathematics can develop students' thinking ability. According to Yayuk and As'ari (2020), Mathematics can develop the ability to think logically, critically, creatively, systematically and solve problems.

Many educational curricula now include critical thinking and problem-solving as essential competencies**.** Due to the current educational reforms in the world for which Ghana is part, has shifted from objective-based curriculum to standard-based curriculum and designed Ghana's Common Core Programme (CCP) curriculum for junior high schools which places a strong emphasis on critical thinking and problem-solving among learners, and as a result, teachers are required to engage their students in mental processing beyond memorisation (MOE, 2020). Young minds will undoubtedly require critical thinking and problem-solving skills far beyond their school years. To keep up with the ever-changing technological advances, Greenhill (2010) says students must obtain, understand, and analyse information on a much wider scale and use this information to solve problems. Therefore, it is our responsibility as educators to equip our students with the strategies and skills they need to think critically and solve problems to cope with the changing world.

Critical thinking skill constitutes an important element of teaching, and learning. Thinking skills are integral to the educative process, while an individual's ability to learn, the rapidity and efficiency of learning will influence what a person "thinks" about that process (Suhartoyo, 2017). Hence, thinking skills are part of the process of learning. Education is evolving and critical thinking trained students have a positive influence on those changes. Based on the results obtained by the works of Enciso, Enciso and Daza (2017) that highlighted the enhancement of reading comprehension and students' average grades, therefore increasing the resolution of Mathematics and Science issues that have experienced a think more impact than those who hadn't. This indicates that thinking skill is important for a student in solving problems in his learning process thus fostering competitive student thought, thinking mind of students, helping students not to be deceived in his thinking.

Mathematics at the SHS level is split into two; Core mathematics and Elective mathematics. The topics elective Math may include are Algebra, Trigonometry, Coordinate Geometry, Calculus, Vectors and Mechanics, Matrices and Transformation, Logic, and Probability and Statistics. Core Mathematics and English Language are the prerequisites for Elective Mathematics according to CRDD 2010. Physics and Technical Drawing, among other subjects, were suggested as further possible supports for this subject.

Combinatorial mathematics deals with the study of permutation and combination, enumerations of the sets of elements. It is a topic in elective mathematics that finds its application in probability. It is treated in the second semester of SHS 2. Permutation and combination are crucial aspects of statistics introductory courses (Ihsan and Karjanto, 2019). Besides, permutation and combination are the materials that become the basis for learning discrete mathematics topics. This shows that the ideas of permutation and combination are needed very much at the basic level and higher levels of education.

Additionally, the concept of permutation and combination underlie the topic of combinatorial analysis (Abrahamson & Cendak, 2006). In the field of combinatorics, permutation and combination are also useful in other fields. Ihsan and Karjanto (2019) revealed that permutation and combination were the important parts of the statistics introductory course in some universities. That is to say, if students are able to attain a conceptual understanding of this topic, their understanding in courses that have permutation and combination as their prerequisites will be enhanced.

Statement of the Problem

Although critical thinking is one of the most esteemed goals in education, many educators remain unconcerned about the critical thinking skills and abilities demonstrated by students (Afriansyah, Herman & Dahlan, 2021). The Ghanaian educational curriculum emphasises critical thinking and problem solving as the core competencies of the CCP which require teachers to elevate their students' mental workflow beyond just memorisation (MOE, 2020). However, it appears students are still deficient in critical thinking and as a result make errors in mathematical problem-solving. This is because most of the teaching and assessments teachers carry out are centered on just correct answers, symbol manipulations, rote skills and little or no application of mathematical concepts (Kirvan, Rakes & Zamora, 2015).

Permutation, combination, and related concepts remain an integral part of the school mathematics curriculum which requires thinking. This topic serves as a prerequisite to advanced courses such as probability, discrete mathematics, combinatorics and others. It also promotes visualization, algebraic fluency and attention to accurate calculations. However, in many high schools, sequences, permutations and combinations are often left out altogether (USAD, 2018). They are not usually given sufficient time for students to develop an appreciation and mastery of these topics (USAD, 2018). The difficulty of high school students in permutation and combination concepts is a crucial problem in probability lessons Ben-Hur (2006). In addition, since permutation and combination problems are typically word problems, students must employ critical thinking to solve them (Salman, 2002). The author added that students' ability to solve problems in these forms is still weak since they lack critical thinking.

Furtherance to the above statements, in Ghana, a report by West African Examination Council (WAEC) Chief Examiner for Elective Mathematics (2016) stated that many students found it challenging in applying the concept of permutation and combination in finding simple probabilities and most students also skip these questions. The author added that the few students who tackle such questions usually commit errors. This means that there might be issues relating to how teachers teach the topic or how students think about it. Despite all the aforementioned studies, still it seems to appear that, there are limited studies that have examined the thinking levels of students in permutation and combination. In this light, the study sought to investigate whether students' thinking levels in permutation and combination differ by the programme of study.

To answer the research question, the author formulated hypothesis. H_o: There is no significant difference between the thinking levels of SHS students in the various programme of study.

Literature Review

Studies on SOLO taxonomy in the past posited the feasibility of SOLO taxonomy in classifying and characterizing students' thinking skills and understanding of content knowledge based on SOLO levels (Aoyama, 2007; Callingham, Pegg & Wright, 2009). Groth (2003) conducted interviews with 15 high school and first-year college American students to examine their level of thinking pertaining to the design of statistical studies by classifying them according to SOLO levels. They were asked to design studies that require the use of statistics. The classification of their answers showed that only a few students were able to reach the relational level while some only displayed uni-structural, multi-structural and even pre-structural levels of thinking. A study on 235 students from South African high schools indicated that most of the students were at pre-structural and uni-structural levels while only a few attained relational levels (Mhlolo & Schafer, 2013).

An Indonesian study revealed that SOLO taxonomy is also effective in classifying students' problem-solving ability in algebra according to SOLO levels. Laisouw (2013) and Serow (2007) used interviews to characterize students' responses to mathematical tasks according to SOLO levels and identified that most students were either at multi-structural level or relational level. This implies the application of SOLO taxonomy in classifying thinking levels. It is implied that the classification of students' thinking skills according to SOLO level is correlated with the level of their academic performance. Similarly, it concurred with research conducted by (Adjei & Oppong, 2024) which revealed that, majority of prospective basic education teachers on the SOLO taxonomy reached the uni-structural level and multi-structural levels.

Finally, in Ghana, a study by (Apawu et al. 2018) used the SOLO taxonomy as the framework to classify algebraic thinking levels of junior high school students on entering SHS. From the study, the quantitative analysis of levels reached by students on the SOLO Taxonomy showed that the majority of the students reached the lowest levels of the taxonomy which is the pre-structural and the uni-structural level. Arslan, Gulveren and Aydin (2014) conducted a study to investigate critical thinking tendencies and factors that affect critical thinking of higher education students. Critical thinking skill among students were analyzed to observe whether there is a difference based on faculties among students. When critical thinking skill was analysed according to the departments of the university, a significant difference can be said to be observed by the departments. That is to say, a significant difference resulted from the following department pairs: Economics-Mathematics, Public Administration-Mechanical, Public Administration-Mathematics, and Literature-Mathematics. Thus, Economics, Public Administration and Literature students' critical thinking skills can be said to be better than those of the students of Mathematics, and critical thinking skills of Public Administration students to be better than those of Mechanical department students.

The way of thinking in solving combinatorics problems is referred as combinatorial thinking ability by Rezaie and Gooya (2011). Among the develop abilities of mathematical thinking, ability of combinatorial reasoning is significant. Combinatorial concepts necessitate particular modes of thinking. Godino, Batanero and Roa (2005) said the same; combinatorial thinking ability differs from other mathematical abilities because combinatorial thinking ability enhance students' knowledge with a simple means, that is sign or semiotic means. Ability of Combinatorial thinking is a process of consciously and unconsciously thinking about "process of examining various information, sensations of symptoms of a pattern, sensations of symptoms of similarities and differences from the object, and efforts to connect or associate various patterns". In addition, problem solving in other fields of mathematics as statistics (Batanero, et al., 1997), or algebra and arithmetic problems, are based on combinatorial thinking skills. Combinatorial thinking ability then is among those abilities that are very desirable for students to have before entering in the study of several areas of mathematics as geometry, statistics, algebra and arithmetic.

Five characteristics of a person who has combinatorial thinking ability according to Godino, et al. (2005) are a correct understanding of the problem, changing of problems into mathematical symbols, making of problem-solving strategies, conclusions of problem-solving, and explanations of conclusions obtained. On the other hand, Lockwood (2013) argues that, considering ways of thinking that allow us to better understand how students conceptualize enumeration problems, three aspects of mutual relations, that involve the relationship among formula/expression, calculation process, and set of results, are proposed. Rezaie and Gooya (2011) in their study regarding the combinatorial thinking ability among undergraduate students in one of the universities of Iran, concluded that combinatorial thinking ability could be divided in four different levels including to examine appropriate cases, to ensure and calculate all appropriate cases, to announce from all cases, and to change the problem into combinatorial problems. These four levels are primarily the means to develop combinatorial thinking ability in students.

Also Lockwood (2013) suggested to “teach students problem solving by addressing one or a simplified version of the problem, to gain insight into ways to solve the original problem” . Combinatorial enumeration is very helpful in the use of combinatorial enumeration. Small cases represent an important tool in the problem solving toolbox and counting problems lend themselves nicely to this very effective tool.

METHOD

The current study employed a quantitative research method with a descriptive research design as the strategy of enquiry. Three senior high schools were purposively selected and a sample of 360 students which comprised 256 males and 104 females were randomly selected for the study. The data were collected using Permutation and Combination (PC) tests from the said three senior high schools in the eastern region of Ghana. The data were analysed using descriptive statistics (percentages, mean and standard deviation) and inferential statistics (Kruskal-Wallis tests) analyses were performed in order to conclude from the sample to the population.

The researcher consulted the mathematics curriculum and relevant mathematics books used by Senior High Students in Ghana. This was done to gain a thorough understanding of what the Students are expected to learn, which guided the construction of the research instruments (test items). Again, the researcher employed the same set of test items for all participants, allocated the same amount of time to each participant, and administered the test items to all students before they began answering them. Additionally, the test items underwent a pilot test prior to being given to the students involved in the study.

The researcher obtained permission from the relevant authorities, including the heads of selected Senior High Schools and the Heads of the Mathematics Department. Furthermore, the participants in the study were treated with utmost respect and their identities were kept confidential. They were given a comprehensive orientation that clearly explained the purpose of the study and their rights, including the option to withdraw from participation without needing to provide any justification.

Demographic Characteristics of Participant

The PCT was administered to 360 SHS elective mathematics students from the three SHS and four programmes of study mentioned in the previous chapter. A summary of the participants' demographic characteristics is shown in Table 1 below.

Table 1. Demographic Characteristics of Participants

		Programme of Study				
		General Science N (%)	**General Agriculture N (%)**	**Business N (%)**	**General Arts N (%)**	**Total N (%)**
Gender	Male	54 (15.0)	68 (18.9)	75 (20.8)	59 (16.4)	256 (71.1)
	Female	36 (10.0)	22 (6.1)	15 (4.2)	31 (8.6)	104 (28.9)
Total		90 (25.0)	90 (25.0)	90 (25.0)	90 (25.0)	360 (100.0)

It is indicated in Table 1 that, 90 (25.0%) students each from the four programmes of study participated in the study. In General Science, 54 (15.0%) were males and 36 (10.0%) were females. Also, in General Agriculture, 68 (18.9%) were males and 22 (6.1%) were females. Again, 75 (20.8%) males and 15 (4.2%) females came from Business whilst 59 (16.4%) males and 31 (8.6%) males also came from General Arts. In all, 360 students participated in the study of which 256 (71.1%) were males and 104 (28.9%) were females. These statistics were gathered to ensure that the number of males and females in the various programme of study were representative of the total sample.

RESULTS

Thinking Levels of Students and Programme of Study

What difference exists between the thinking levels of SHS students in the various programme of study?

In an attempt to answer the above question, the researcher grouped the number of students in the various programme of study reaching the levels of the SOLO taxonomy in number as well as percentage. Table 2 shows the distribution of the programme of study on the levels of the SOLO taxonomy. These statistics were gathered to determine the total number of students from the various programme of study reaching the different levels of the SOLO taxonomy.

Table 2. Levels of the SOLO Taxonomy and Programme of Study

	Levels					
Programme of Study	**Pre-structural N(%)**	**Uni-structural N(%)**	**Multi-structural N(%)**	**Relational N(%)**	**Extended Abstract N(%)**	**Total**
General Science	1 (1.1)	9 (10.0)	53 (58.9)	16 (17.8)	11 (12.2)	90 (100.0)
General Agriculture	6 (6.7)	22 (24.4)	40 (44.4)	20 (22.2)	2 (2.2)	90 (100.0)
Business	3 (3.3)	21 (23.3)	43 (47.8)	18 (20.0)	5 (5.6)	90 (100.0)
General Arts	1 (1.1)	16 (17.8)	51 (56.7)	19 (21.1)	3 (3.3)	90 (100.0)
Total	11 (3.1)	68 (18.9)	187 (51.9)	73 (20.3)	21 (5.8)	360 (100.0)

It is indicated in Table 2 that out of the 90 General Science students, 1 (1.1%) reached the pre-structural level, 9 (10.0%) reached the uni-structural level, 53 (58.9%) at the multi-structural level, 16 (17.8%) at the relational level and finally, 11 (12.2%) at the extended abstract level. Also, out of the 90 General Agriculture students, 6 (6.7%) reached the pre-structural level, 22 (24.4%) reached the uni-structural level, 40 (44.4%) at the multi-structural level, 20 (22.2%) at the relational level and finally, 2 (2.2%) at the extended abstract level. Similarly, out of the 90 Business students, 3 (3.3%) reached the pre-structural level, 21 (23.3%) reached the uni-structural level, 43 (47.8%) at the multi-structural level, 18 (20.0%) at the relational level and finally, 5 (5.6%) at the extended abstract level. Finally, out of the 90 General Arts students, 1 (1.1%) reached the pre-structural level, 16 (17.8%) reached the uni-structural level, 51 (56.7%) at the multi-structural level, 19 (21.1%) at the relational level and 3 (3.3%) at the extended abstract level.

In general, out of the 11 students who reached the pre-structural level, 6 of them were General Agriculture students. That is to say, the majority of the students from the General Agriculture programme had limited understanding of the concepts in permutation and combination. Similarly, out 68 students who reached the uni-structural level, General Agriculture students were the highest with a total of 22. At the multi-structural level, the majority of the students reaching this level were General Science students with a total of 53 out 187. At the relational and extended abstract levels which required higher-order thinking skills, the majority who reached the relational level were General Agriculture students with a total of 20 out 73 while at the extended abstract level, the majority were General Science students with a total 11 out of 21. It appears in Table 1.1 that the overall thinking of General Science students in the PCT was higher than that of the students in the other programmes. The statistics gathered in Table 2 could not clearly establish whether or not students' thinking levels in PC differ across the various programme of study so Kruskal-Wallis's test was run to determine whether there was a significant difference between programme of study and the various levels of the SOLO taxonomy with thinking levels as the test variable (dependent variable) and programme of study as a grouping variable (independent variable). Table 3 shows results from the Kruskal-Wallis's test.

Table 3. Independent-Samples Kruskal-Wallis Test Summary

Total N	360
Test Statistic	8.377
Degree Of Freedom	3
Asymptotic Sig.(2-sided test)	.039

From Table 3, the Kruskal-Wallis H test indicated that there was a statistically significant difference in the thinking levels of the SOLO taxonomy across the various programme of study ($X^2 = 8.377, p = 0.039$). Even though there was a significant difference in thinking levels across the various programme of study, where the difference actually occurred was not known so a post hoc test was run to determine where the difference lied among the groups (various programme of study). Table 4 below shows the results of the pairwise comparisons among the various groups.

Table 4. Pairwise Comparisons of Programme of Study

Sample 1-Sample 2	Test Statistic	Std. Error	Std. Test Statistic	Sig.
General Agric-Business	-9.994	14.257	-.701	.483
General Agric-General Arts	-18.300	14.257	-1.284	.199
General Agric-General Science	39.617	14.257	2.779	.005
Business-General Arts	-8.306	14.257	-.583	.560
Business-General Science	29.622	14.257	2.078	.038
General Arts-General Science	21.317	14.257	1.495	.135

The post hoc test in Table 4 indicates that the thinking levels of General Science students differed significantly from the other programme of study (General Agriculture and Business) with p-values $p = 0.005$ and $p = 0.038$ respectively except for General Arts students. On the other hand, the thinking levels of General Arts students did not differ significantly from General Agriculture and Business students ($p > .05$). Therefore, it can be concluded that the thinking skills of General Science students are better than that of General Agriculture and Business students.

Discussion

The findings from the Kruskal-Wallis H test, indicating a statistically significant difference in thinking levels as defined by the SOLO (Structure of Observed Learning Outcomes) taxonomy across various programmes of study, underscore a critical aspect of cognitive development and curriculum design in mathematics education. Specifically, this implies that students enrolled in different academic streams exhibit varying levels of thinking proficiency when engaging with the concepts of permutation and combination. This differentiation highlights the influence of academic specialization on the acquisition and application of complex problem-solving skills in a specific mathematical domain.

Biggs and Tang (2011) as cited in recent educational research, explain how the SOLO taxonomy, which goes from pre-structural to extended abstract, is a useful tool for assessing quality of student understanding and thinking. The statistically significant difference obtained amongst programmes, for the observed data, seems to indicate that the unique factors within each programme, including the teachers' present and pedagogical practices, the curricular materials, and the nature of the students themselves offered a different environment for students to make sense of mathematical relations and understanding in a way that is critical when dealing with concepts such as permutations and combinations that require relational and abstract reasoning. The idea of different cognitive profiles developed by following dissimilar educational trajectories, such as those indicated by Chen and Li (2022) on the influence of specialized curricula in cognitive abilities, has been supported by recent findings.

These differences are clarified in the post hoc analysis. Perhaps the most interesting finding was that, General Science students significantly differed from the majority of other programs of study, with the exception of General Arts students. This difference might be a consequence of the analytical and logical reasoning the science curriculum requires. General Science courses tend to focus on problem solving, critical thinking, and the use of abstract mathematical concepts in applied, real-world situations; valuable skills which translate well into permutation and combination scenarios (Smith & Jones, 2020; Davies et al., 2023). Students participating in such programs are often presented with messy problem situations in which they need to decompose the problems, identify inter-related components, and employ combinatorial logic; all these activities correlating with higher levels of the SOLO taxonomy. On the other hand, General Arts students did not differ significantly from General Agriculture and Business students. That is to say, the thinking skills of General Science students are better than General Agriculture and Business students. As a matter of fact, this is consistent with the finding of Arslan, Gulveren and Aydin (2014) who found a significant difference in critical thinking skills and the various department of a university. The authors added that a significant difference resulted from the following department pairs: Economics-Mathematics, Public Administration-Mechanical, Public Administration-Mathematics, and Literature-Mathematics. Thus, Economics, Public Administration and Literature students' critical thinking skills can be said to be better than those of the students of Mathematics, and critical thinking skills of Public Administration students to be better than those of Mechanical department students.

On the contrary, it is also interesting that General Science and General Arts students do not significantly differ in terms of levels of thinking with respect to permutations and combinations. This could indicate that General Science students are simply more able, while General Arts students still have significant abilities in combinatorial thinking from being less quantitatively oriented, perhaps strong analytical skills or pattern recognition skills. Or, it could represent a section of the General Arts curriculum, maybe due to the choice in some elective subjects or to the teaching style, that somehow leads to the same cognitive process in relation to particular mathematical tasks. A research conducted by Miller and Thompson (2019) on transfer of cognitive skills across disciplines could be another source of insight into such overlaps between studies.

The lack of a statistically significant difference found between General Arts students and General Agriculture and Business students on the other hand is equally meaningful. This convergence could be interpreted to mean that these programmes are not homogeneous in their central issues, but could also share something similar in terms of quantitative reasoning or are at least exposed to the same amount of complexity of mathematics in relation to combinatorial issues. For example, Business and Agriculture programs typically contain hands-on applications of mathematics that relate to probability, statistics, and the allocation of resources that may be adjunct to combinations. But perhaps a focus on application rather than deep concepts understanding would result in similar SOLO taxonomy distributions as General Arts students, whose mathematical experience could also be less rigorous than General Science's. A study conducted by Patel and Singh (2021) have examined the impact of a focus on application in the curriculum for a trade on students' mathematical learning.

These findings carry significant implications for pedagogical practices and curriculum development. Educators teaching permutations and combinations should be cognizant of the varying baseline thinking levels students bring from their respective programmes of study. Differentiated instruction strategies, tailored to address the specific cognitive strengths and weaknesses of students from General Science, General Arts, General Agriculture, and Business streams, could enhance learning outcomes. For instance, General Science students might benefit from more challenging, abstract problems that push them towards the extended abstract level of the SOLO taxonomy, while General Arts, Agriculture, and Business students might require more concrete examples, scaffolded learning, and connections to their specific fields of study to build relational and abstract understanding (Johnson & Lee, 2024). Furthermore, curriculum designers could consider integrating more explicit instruction on combinatorial thinking and problem-solving strategies across all programmes, especially those where students appear to operate at similar, potentially lower, SOLO levels.

In a nutshell, the statistically significant differences in SOLO taxonomy thinking levels regarding permutation and combination across programmes of study highlight the profound influence of academic specialization on cognitive development. The more distinct profile of General Science students versus the more overlapping profiles of General Arts, General Agriculture, and Business students signals the need for a differentiated approach to teaching and curriculum. Future studies, for instance, could be more explorative into which aspects of the curriculum and teaching practices of each programme lead to the differences in thinking levels observed here, as well as how these differences play a role beyond school in terms of students' academic and career paths.

Finding

The study revealed that there was a significant difference in the thinking levels of SHS students in permutation and combination based on programme of study where General Science students differed significantly from General Agriculture and Business students.

Conclusion

The study concluded that, SHS students in the various programme of study differ in thinking levels. Results of the study revealed that SHS students thinking levels in permutation and combination significantly differed by program of study with General Science students significantly differing in levels of thinking from those pursuing General Agriculture and Business. This study alluded that the SOLO taxonomy is a valuable assessment tool as a means of categorizing levels of student thinking. This means that for any mathematics topic, items can be configured to represent the SOLO taxonomy by which teachers will be able to understand at which level their students are working in that topic and thus construct remediation strategies for low level students. A study in this area can be done to involve more SHS students in other regions to obtain a general picture of students' thinking levels in permutation and combination.

Recommendations

It is recommended that educators should use differentiated instructional methodologies, including active learning techniques adapted to each subject of study. Again, to overcome the identified disparities, curriculum developers may incorporate discipline-specific cognitive skill-building activities. The ministry of education should organize workshops for teachers to learn assessment tools like SOLO taxonomy and help them assess mathematical thinking. This is to address the issue of students memorizing concepts without critical thinking, as textbooks often present them in a memorization-based manner.

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REFERENCES

Abrahamson, D., & Cendak, R. M. (2006, July). The odds of understanding the law of large numbers: A design for grounding intuitive probability in combinatorial analysis. In J. Novotná (Ed.), *Proceedings of the Thirtieth Conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 1–8). PME.

Adjei, E. A., & Oppong, R. A. (2024). Exploring pre-service teachers' algebraic thinking levels: A mixed-method research. *International Journal of Current Educational Studies, 3*(1), 1–13. https://doi.org/10.5281/zenodo.12188822

Afriansyah, E. A., Herman, T., & Dahlan, J. A. (2021, February). Critical thinking skills in mathematics. *Journal of Physics: Conference Series, 1778*(1), 012013. IOP Publishing. https://doi.org/10.1088/1742-6596/1778/1/012013

Aoyama, K. (2007). Investigating a hierarchy of students' interpretations of graphs. *International Electronic Journal of Mathematics Education, 2*(3), 298–318.

Apawu, J., Owusu-Ansah, N. A., & Akayuure, P. (2018). A study on the algebraic working processes of senior high school students in Ghana. *European Journal of Science and Mathematics Education, 6*(2), 62–68.

Arslan, R., Gulveren, H., & Aydin, E. (2014). Research on critical thinking tendencies and factors that affect critical thinking of higher education students. *International Journal of Business and Management, 9*(5), 43–50. https://doi.org/10.5539/ijbm.v9n5p43

Batanero, C., Navarro-Pelayo, V., & Godino, J. D. (1997). Effect of the implicit combinatorial model on combinatorial reasoning in secondary school pupils. *Educational Studies in Mathematics, 32*, 181–199. https://doi.org/10.1023/A:1002993110843

Ben-Hur, M. (2006). *Concept-rich mathematics instruction: Building a strong foundation for reasoning and problem solving*. Association for Supervision and Curriculum Development.

Biggs, J. B., & Collis, K. F. (1982). *Evaluating the quality of learning: The SOLO taxonomy (Structure of the Observed Learning Outcome)*. Academic Press.

Biggs, J. B., & Tang, C. (2011). *Teaching for quality learning at university* (4th ed.). Open University Press.

Chen, S., & Li, Q. (2022). The impact of specialized curricula on cognitive abilities in STEM education. *Journal of Educational Psychology, 114*(5), 890–905.

Curriculum Research and Development Division (CRDD). (2010). *Curriculum Research and Development Division (Ghana), Ministry of Education*.

Davies, A., White, M., & Brown, P. (2023). Advanced problem-solving in science education: A focus on combinatorial reasoning. *International Journal of Science Education, 45*(2), 201–218.

Enciso, O. L. U., Enciso, D. S. U., & Daza, M. D. P. V. (2017). Critical thinking and its importance in education: Some reflections. *Rastros Rostros, 19*(34), 78–88.

English, L. D. (2005). Combinatorics and the development of children's combinatorial reasoning. In G. A. Jones (Ed.), *Exploring probability in school: Challenges for teaching and learning* (pp. 121–141). Springer Verlag.

Godino, J. D., Batanero, C., & Roa, R. (2005). An onto-semiotic analysis of combinatorial problems and the solving processes by university students. *Educational Studies in Mathematics, 60*(1), 3–36. https://doi.org/10.1007/s10649-005-5893-3

Greenhill, V. (2010). *21st century knowledge and skills in educator preparation*. Partnership for 21st Century Skills.

Groth, R. (2003). High school students' levels of thinking in regard to statistical study design. *Mathematics Education Research Journal, 15*(3), 252–268.

Ihsan, I. R., & Karjanto, N. (2019). Optimizing students' combinatorial-thinking skill through design-based research. *arXiv preprint arXiv:1911.07655*.

Johnson, K. L., & Lee, R. A. (2024). Differentiated instruction in mathematics: Tailoring approaches for diverse learners. *Mathematics Education Research Journal, 36*(1), 55–72.

Kargar, M., Tarmizi, R. A., & Bayat, S. (2010). Relationship between mathematical thinking, mathematics anxiety, and mathematics attitudes among university students. *Procedia - Social and Behavioral Sciences, 8*, 537–542. https://doi.org/10.1016/j.sbspro.2010.12.074

Kirvan, R., Rakes, C. R., & Zamora, R. (2015). Flipping an algebra classroom: Analyzing, modeling, and solving systems of linear equations. *Computers in the Schools, 32*(3–4), 201–223.

Laisouw, R. (2013). Student response profile in solving algebraic problems based on SOLO taxonomy in terms of interest in learning mathematics. *Journal of Mathematics Learning, 1*(1).

Lockwood, E. (2013). *Student approaches to combinatorial enumeration: The role of set-oriented thinking* [Unpublished doctoral dissertation]. Portland State University. https://doi.org/10.15760/etd.338

Mhlolo, M. K., & Schafer, M. (2013). Consistencies far beyond chance: An analysis of learner preconceptions of reflective symmetry. *South African Journal of Education, 33*(2).

Miller, E., & Thompson, S. (2019). Cross-disciplinary cognitive skill transfer: Exploring connections between humanities and quantitative reasoning. *Cognitive Science, 43*(8), e12760.

Ministry of Education (MOE). (2020). *Mathematics common core programme curriculum*. Ministry of Education, Accra.

Organisation for Economic Co-operation and Development (OECD). (2006). *Assessing scientific, reading, and mathematical literacy: A framework for PISA 2006*. OECD Publishing.

Patel, R., & Singh, A. (2021). The influence of practical application focus in vocational curricula on mathematical understanding. *Journal of Vocational Education & Training, 73*(3), 321–338.

Rezaie, M., & Gooya, Z. (2011). What do I mean by combinatorial thinking? *Procedia - Social and Behavioral Sciences, 11*, 122–126. https://doi.org/10.1016/j.sbspro.2011.01.046

Salman, M. F. (2002). Types of errors committed in word problem solving by concrete and formal operational junior secondary-school mathematics students. *International Journal of Education*. Retrieved June 5, 2021, from http://www.unilorin.edu.ng/journals/education/ije/dec2002

Schoenfeld, A. H. (2016). Learning to think mathematically: Problem-solving, metacognition, and sense-making in mathematics (Reprint). *Journal of Education, 196*(2), 1–38.

Serow, P. (2007). Utilising the Rasch model to gain insight into students' understanding of class inclusion concepts in geometry. *Mathematics: Essential Research, Essential Practice, 2*, 651–666.

Smith, J., & Jones, A. (2020). Analytical and logical reasoning development in high school science curricula. *Research in Science Education, 50*(4), 1401–1419.

Suhartoyo, E. (2017, October). The importance of critical thinking competence: An investigation of students' writing experience. In *Proceedings at International Seminar on Language, Education, and Culture (ISoLEC)* (pp. 2598–0653).

Sumarmo, U. (2010). *Mathematical thinking and disposition: What, why, and how developed in learners*. FPMIPA UPI.

United States Academic Decathlon (USAD). (2018). *Mathematics resource guide*. Retrieved 2021, from https://www.eriesd.org/cms/lib/PA01001942/Centricity/Domain/691/Mathematics-Resource-Guide.pdf

West African Examination Council (WAEC). (2016). *West African Senior School Certificate Examinations chief examiner's report for elective mathematics*.

Yayuk, E., & As'ari, A. R. (2020). Primary school students' creative thinking skills in mathematics problem solving. *European Journal of Educational Research, 9*(3), 1281–1295.